European Journal of Mechanics B/Fluids 34 (2012) 64–69

Contents lists available at SciVerse ScienceDirect

European Journal of Mechanics B/Fluids

journal homepage: www.elsevier.com/locate/ejmflu

Stokes flow through a microchannel obstructed by a vertical plate

Seok-Hyun Yoon, Jae-Tack Jeong *

School of Mechanical Systems Engineering, Chonnam National University, 300 Yongbong-Dong, Gwangju 500-757, Republic of Korea

ARTICLE INFO

Article history:
Received 24 August 2011
Received in revised form
13 February 2012
Accepted 1 March 2012
Available online 5 March 2012

Keywords:
Microchannel
Stokes flow
Blockage factor
Drift velocity
Poiseuille flow
Pressure drop

ABSTRACT

Two-dimensional Stokes flow through a microchannel obstructed by a vertical plate is investigated on the basis of Stokes approximation. The plate translates along the center-line of the channel and plane Poiseuille flow exists upstream and downstream from the plate. The Stokes flow is analyzed analytically using the eigenfunction expansion and the point collocation method. The stream function and the pressure distribution of the flow field are obtained for an arbitrary translating velocity of the plate and arbitrary magnitude of the Poiseuille flow. The force exerted on the plate and the pressure drop induced by the plate are calculated as functions of blockage factor. From the results, the drift velocity of the plate, for which the force exerted on the plate vanishes and the plate translates freely in the Poiseuille flow, is determined. The drift velocity is slightly lower than the mean velocity of the Poiseuille flow component projected by the plate, and induced pressure drop due to the drifting plate in the Poiseuille flow is quite small.

© 2012 Elsevier Masson SAS. All rights reserved.

1. Introduction

The development of bio-MEMS technology has shed light upon the slow viscous characteristics of microchannels. Microchannels are the most basic elements of microfluidic devices [1–4]. Hemodynamics requires the knowledge and understanding of the flow characteristics of freely flowing matters. For example, blood coagulation and intravascular material cause embolus (blood clot). The blood flow in a blood vessel may be modeled as viscous microchannel flow in a two-dimensional model. Accordingly, the size of flow obstacles and the Reynolds number were used to investigate flows through microchannels [5–8].

Many authors have considered the Stokes flow in the channel which contains various types of obstruction. Wang [6] studied the flow in a channel containing periodic obstructions using eigenfunction expansion and point collocation. Through such methods, Wang investigated the Stokes flow past circular cylinders in the channel. Jeong [9] investigated the Stokes flow of a two-dimensional microchannel using a pair of vertical plates attached to the channel wall and applying slip boundary conditions. Davis [10] computed induced pressure gradient due to a periodic array of wall-attached barriers in a channel flow and considered an equivalent flow with slip boundary condition. Crowdy [11] showed that the frictional slip lengths for Stokes flow are connected with the blockage coefficients for potential flow. Kim [12] analyzed the force exerted on a vertical plate between two parallel plates. Kalita and Gupta [13] numerically obtained the stream function for the flow approaching a square obstacle in the channel.

In this paper, we investigate a two-dimensional slow viscous flow through a microchannel of height $2H$, past a translating obstacle, as shown in Fig. 1. The obstacle modeled by a thin vertical plate of length $2a$ moves in the x-direction with a velocity of U. Far from the obstacle, there exists Poiseuille flow of a mean velocity V. This study is motivated by transportation of a particle in Poiseuille flow in channel, which may appear in the blood flow in the blood vessel or the drug delivery system [14]. The drug transportation velocity is important to increase the drug efficacy and to reduce adverse drug reactions. The flow fields for the parameters 'U', 'V' *and* 'a' are investigated analytically by using Stokes approximation because of the small length scale of the microchannel height and the low velocity of flow. The force exerted on the plate and induced pressure drop due to the plate are determined as functions of the blockage factor a/H. The most interesting result may be drift velocity of the plate when it is swept away by the Poiseuille flow.

2. Method of solution

We evaluate an incompressible viscous flow in a channel containing a vertical flat plate, as shown in Fig. 1. The height of the channel is $2H$, the length of the plate is $2a$ and the thickness of the plate is assumed to be zero. The blockage factor is defined as a/H. A plane Poiseuille flow through the channel exists upstream and downstream of the vertical plate. For convenience, we normalize the length scale of the flow region with H by taking H is equal to 1. Referring to Cartesian coordinates (x, y) defined in Fig. 1, the

* Corresponding author. Tel.: +82 62 530 1673; fax: +82 62 530 1689.
E-mail address: jtjeong@chonnam.ac.kr (J.-T. Jeong).

0997-7546/$ – see front matter © 2012 Elsevier Masson SAS. All rights reserved.
doi:10.1016/j.euromechflu.2012.03.001

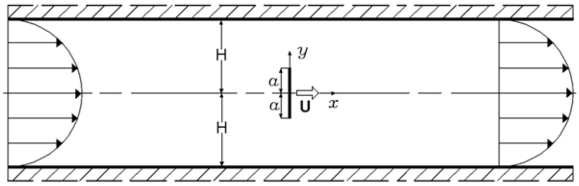

Fig. 1. Geometry of the two-dimensional microchannel.

channel walls are represented by $-\infty < x < \infty, y = \pm 1$ and the plate is represented by $x = 0, -a \le y \le a$. The plate translates in the x-direction with velocity U and the mean velocity of the Poiseuille flow is V.

When the Reynolds number $\mathrm{Re}_H \equiv \max\{(UH/\nu), (VH/\nu)\}$ is small, the inertia term is negligible in Navier–Stokes equation and the motion of the fluid is governed by the Stokes equation,

$$\nabla \cdot \vec{u} = 0, \tag{1}$$

$$\nabla p = \mu \nabla^2 \vec{u}, \tag{2}$$

where $\vec{u} = (u, v)$ is the velocity, μ is the coefficient of viscosity and p is the pressure. For a two-dimensional incompressible flow, we introduce a stream function $\psi(x, y)$ such that

$$u = \frac{\partial \psi}{\partial y}, \qquad v = -\frac{\partial \psi}{\partial x}. \tag{3}$$

Substituting Eq. (3) into Eq. (2) and canceling $p(x, y)$, $\psi(x, y)$ satisfies the biharmonic equation, [15]

$$\nabla^4 \psi = \left(\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} \right)^2 = 0. \tag{4}$$

Since the biharmonic equation (4) is linear, the resultant flow must be zero when superposed by the reverse flow. Therefore, the flow must be symmetric about $x = 0$. Moreover, the geometry and boundary conditions are symmetric about the line $y = 0$ and the streamfunction must be anti-symmetric about $y = 0$. Hence,

$$\psi(x, y) = \psi(-x, y) = -\psi(x, -y), \tag{5}$$

$$p(x, y) = -p(-x, y) = p(x, -y). \tag{6}$$

Hence, it is sufficient to consider the flow region of $x \ge 0$, $0 \le y \le 1$ only. Considering no-slip boundary conditions for $\psi(x, y)$ on the channel wall and the moving plate,

$$\psi(x, 1) = V, \quad -\infty < x < \infty \tag{7a}$$

$$\psi_y(x, 1) = 0, \quad -\infty < x < \infty \tag{7b}$$

$$\psi_x(0, y) = 0, \quad -1 \le y \le 1 \tag{7c}$$

$$\psi(0, y) = Uy, \quad -a \le y \le a. \tag{7d}$$

Additionally, considering the symmetry condition (6) and continuity of stress in the flow field, we get

$$p(0, y) = 0, \quad a < |y| \le 1 \tag{7e}$$

and the Poiseuille flow condition far from the plate is

$$\psi(x, y) \to \frac{V(3y - y^3)}{2}, \quad \text{as } |x| \to \infty. \tag{7f}$$

Considering the symmetry of the flow and the boundary conditions, we set the stream function $\psi(x, y)$ satisfying Eq. (4) for $0 \le x < \infty$ as

$$\psi(x, y) = \frac{V(3y - y^3)}{2} + \int_0^\infty A(\xi) \cos \xi x \cdot (\cosh \xi \sinh \xi y - y \sinh \xi \cosh \xi y) d\xi + \sum_{n=1}^{\infty} B_n \sin n\pi y \cdot \{e^{-n\pi x}(n\pi x + 1)\}, \tag{8}$$

where the first term in the right-hand side $V(3y - y^3)/2$ represents the Poiseuille flow of mean velocity V in channel, and $A(\xi)$, B_n $(n = 1, 2, 3, \dots)$ are unknowns to be determined. Then, the stream function in Eq. (8) satisfies governing Eq. (4) and boundary conditions (7a), (7c) and (7f).

Using Eq. (8), boundary condition (7b) can be written as follows;

$$\int_0^\infty A(\xi)(\xi - \sinh \xi \cosh \xi) \cos \xi x d\xi + \sum_{n=1}^{\infty} (-1)^n B_n n\pi e^{-n\pi x}(n\pi x + 1) = 0. \tag{9}$$

Applying the Fourier cosine integral and residue theorem [16] to Eq. (9), we can express $A(\xi)$ in terms of B_n. Substituting $A(\xi)$ into Eq. (8), we obtain, for $0 \le x < \infty$,

$$\psi(x, y) = \frac{V(3y - y^3)}{2} - \sum_{n=1}^{\infty} (-1)^n 4n^4 \pi^4 B_n \times \mathrm{Im} \sum_{m=1}^{\infty} \frac{\cosh \xi_m \sinh \xi_m y - y \sinh \xi_m \cosh \xi_m y}{(n^2\pi^2 + \xi_m^2)^2 \sinh^2 \xi_m} e^{i\xi_m x}. \tag{10}$$

Here and hereafter, "Re" and "Im" denote the real and the imaginary parts, respectively. The constants ξ_m $(m = 1, 2, 3, \dots)$ in Eq. (10) are complex roots of equation,

$$\xi - \sinh \xi \cosh \xi = 0, \tag{11}$$

with positive real and imaginary parts. If z is a root, then so are $z, \bar{z}$ and $-\bar{z}$. Here, $\bar{z}$ denotes the complex conjugate of z. The roots are ordered so that $0 < \mathrm{Im}\xi_1 < \mathrm{Im}\xi_2 < \mathrm{Im}\xi_3 < \cdots$. The asymptotic value of ξ_m is given by,

$$\xi_m \to \left(m + \frac{1}{4}\right)\pi i + \frac{1}{2} \ln \left[\left(2m + \frac{1}{2}\right)\pi + \left\{ \left(2m + \frac{1}{2}\right)^2 \pi^2 - 1 \right\}^{1/2} \right] \quad \text{for } m \gg 1. \tag{12}$$

Terms in the summation for m in Eq. (10) may be thought as complex eigenfunctions with eigenvalues ξ_m $(m = 1, 2, 3, \dots)$. The series form of Eq. (10) is more efficient than the ordinary series expansion obtained by separation of variables when determining unknown coefficients. To obtain the stream function completely, we must determine the unknowns B_n $(n = 1, 2, 3, \dots)$ in Eq. (10) using the yet unused boundary condition (7d) and (7e).

To use boundary condition (7e), we derive the pressure distribution $p(x, y)$ from Eqs. (2) and (3) as, for $0 \le x < \infty$,

$$p(x, y) = -3\mu V x - \mu \sum_{n=1}^{\infty} (-1)^n 8n^4 \pi^4 B_n \times \mathrm{Re} \sum_{m=1}^{\infty} \frac{\xi_m \cosh \xi_m y}{(n^2\pi^2 + \xi_m^2)^2 \sinh \xi_m} e^{i\xi_m x} - K, \tag{13}$$

where the first term in the right-hand side $-3\mu V x$, which is dominant as $x \to \infty$, represents the linearly decreasing pressure corresponding to the Poiseuille flow of mean velocity V in the channel and the last term $-K$ represents yet unknown pressure change due to the vertical plate. Note that positive K indicates the pressure drop and negative K indicates the pressure rise. Now, boundary conditions (7d) and (7e) can be expressed as,

Download English Version:

https://daneshyari.com/en/article/650592

Download Persian Version:

https://daneshyari.com/article/650592

Daneshyari.com