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Multiphase lattice Boltzmann simulations of buoyancy-induced flow of two immiscible fluids with different viscosities

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ABSTRACT

We study the effects of viscosity differential on buoyancy-induced interpenetration of two immiscible fluids in a tilted channel using a two-phase lattice Boltzmann method implemented on a graphics processing unit. The effects of viscosity differential on the flow structures, average density profiles and front velocities are studied. Relatively stable fingers are observed for high viscosity ratios. The intensity of the interfacial instabilities and the transverse interpenetration of the fluids are seen to increase with decreasing viscosity differential of the fluids.

1. Introduction

A flow problem of considerable industrial and natural importance is the interpenetration of two immiscible fluids initially separated by a partition and suddenly allowed to mix by the action of the gravitational force [1–4]. This problem is frequently referred to as the lock-exchange problem [3,5–7]. This phenomenon plays an important role in the design of chemical and petroleum engineering processes [1,2] as well as for the understanding of various natural systems in oceanography and atmospheric sciences [8]. In the absence of pressure-gradient, buoyancy-induced mixing has been previously studied experimentally by Séon and co-authors [9–12] and numerically by Hallez and Magnaudet [7] and recently by Sahu and Vanka [13].

Fig. 1 shows a schematic of the flow situation. The parameters characterizing the flow evolution in this configuration are density contrast characterized by Atwood number, *At* which is defined as $(\rho_1-\rho_2)/(\rho_1+\rho_2)$, the tilt angle (measured from vertical) and the viscosities of the two fluids. ρ_1 and ρ_2 are the densities of fluids '1' and '2', respectively. In the experiments of Séon et al. [10] three distinct regions of the flow and mixing patterns were observed as the tilt angle was increased. For smaller tilt angles ($\theta < 65°$, θ is the angle with vertical), the velocity of the high and low density fluids (moving in the opposite directions) increased linearly as the tilt angle was increased. The mixing (or interpenetration) of the two fluids could be characterized by a diffusive process described by a diffusion equation with a suitably defined diffusion coefficient. The flow and mixing is influenced by two distinct processes. First, the gravitational force's component along the axis of the channel accelerates the two fluids into each other at comparable velocities depending on the individual densities of the fluids. The interface of the two fluids becomes unstable and gives rise to the Kelvin–Helmholtz (KH) instabilities, and consequent transverse mixing. However, the component of the gravity force normal to the channel axis has an opposite effect by acting to segregate the two fluids. The front velocity (V_f) depends on the local density contrast across the interface, which in turn is decreased from the initial maximum value because of diffusional mixing (smearing) of the interface. Thus, while the KH instabilities decrease the front velocity, the segregation of the two fluids caused by the transverse gravitational force increases the front velocity. For $65° < \theta < 82°$, the flow is characterized by a regime in which the front velocity is nearly constant, with a value approximately $0.7\sqrt{Atgd}$, where g is the gravitational acceleration and d is a characteristic dimension of the confining channel. The region of constant V_f depends on the fluid viscosity (for a given *At* number) with a higher fluid viscosity causing an earlier transition to the constant front velocity limit. This is a result of decreased small scale transverse mixing due to the lower Reynolds number. When the pipe is almost horizontal ($\theta > 82°$) the flow transitions to a third region in which the

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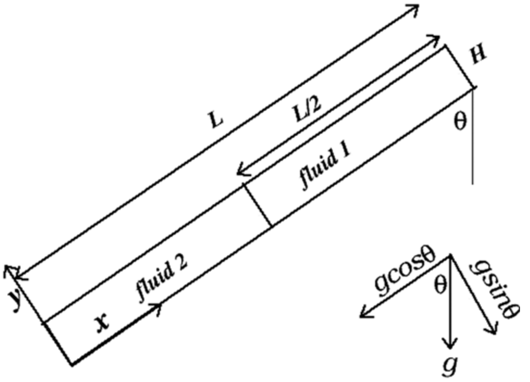

Fig. 1. Schematic diagram of the initial equilibrium configuration of the system. The aspect ratio of the channel is 1:40.

two fluids move as counter-current Poiseuille flows and the front velocity is then determined by the balance between buoyancy and wall friction. In this region, the velocity decreases with tilt angle because of the reduced buoyancy force.

Only a limited number of computational studies have been reported for this flow. Hallez and Magnaudet [7] solved the governing equations with a finite volume method and studied the buoyancy-induced mixing of two fluids in circular, rectangular and square geometries. They found that the vortices which develop during the flow are more coherent and persistent in two than in three dimensions. Consequently, the vortices give rise to more intense mixing and long-lasting flow structures in two-dimensions than in three dimensional geometries. Recently, Sahu and Vanka [13] used a two-phase lattice Boltzmann method (LBM) to simulate the interpenetration of two immiscible fluids in a tilted channel. They investigated the effects of Atwood number, Reynolds number, tilt angle and surface tension in terms of flow structures, front velocities and velocity profiles. They compared result of a typical case with that of a finite volume method using the diffuse interface technique [4] and found good agreement. The results also compared well with the previous experimental results [3,9,10]. In a related work, Zhang et al. [14] studied the effects of surface tension on the development of the Kelvin–Helmholtz instabilities in two-dimensional mixing layer via a similar multiphase approach proposed by He et al. [15].

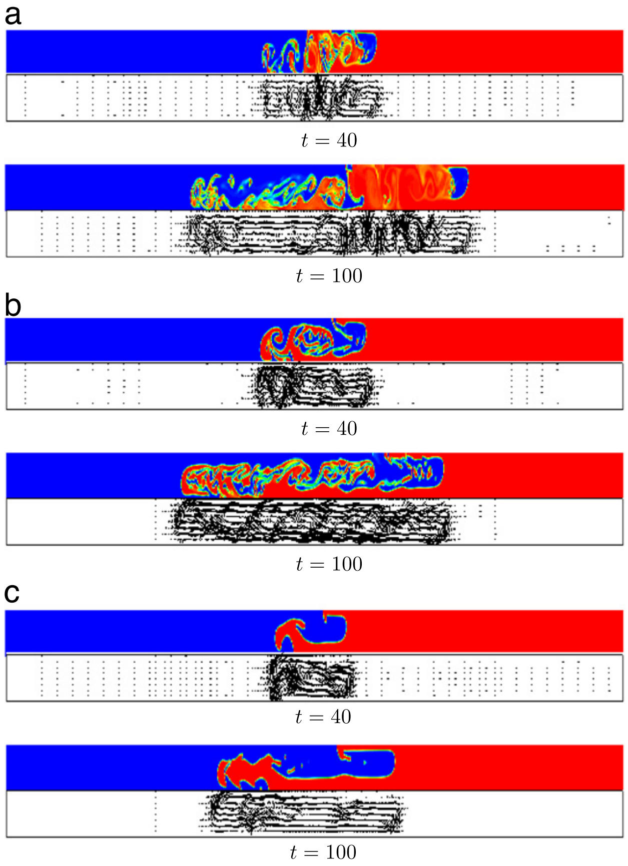

Fig. 3. Spatio-temporal evolutions of the density contours and velocity vector-fields for (a) $m = 0.1$, (b) $m = 1$ and (c) $m = 10$. The rest of the parameter values are Re = 500, At = 0.05, $\kappa = 0$ and $\theta = 30°$.

All the above mentioned studies considered the case of two fluids having equal viscosities. The fluids used in the experiments of Debacq et al. [3] were water and glycerol solutions with the heavier fluid generated by adding $CaCl_2$. The viscosity of the two fluids were changed by varying the concentration of glycerol, but both the fluids were selected to be of the same viscosity. However, the dynamics of the unsteady mixing can be quite different if the viscosities of the two fluids differed significantly from each other. In pressure-driven displacement flows, a similar situation was studied by Sahu et al. [16,17], Taghavi et al. [18]. To the best of our knowledge, no previous studies exist in the literature in which the case of unequal fluid viscosities have been considered in the "lock-exchange" problem. In this paper, we study the "mixing" of two molecularly immiscible fluids in a tilted channel with viscosity ratios, $10 \leq m \leq 0.1$ between the heaver and lighter fluids; $m = 1$ being the case when both the fluids have the same viscosity. We use the same multiphase LBM approach used in our earlier study [13].

The rest of this paper is organized as follows. Section 2 describes the problem considered and gives a brief description of the numerical method used. Since the method has been recently validated and presented in our recent work [13], we briefly

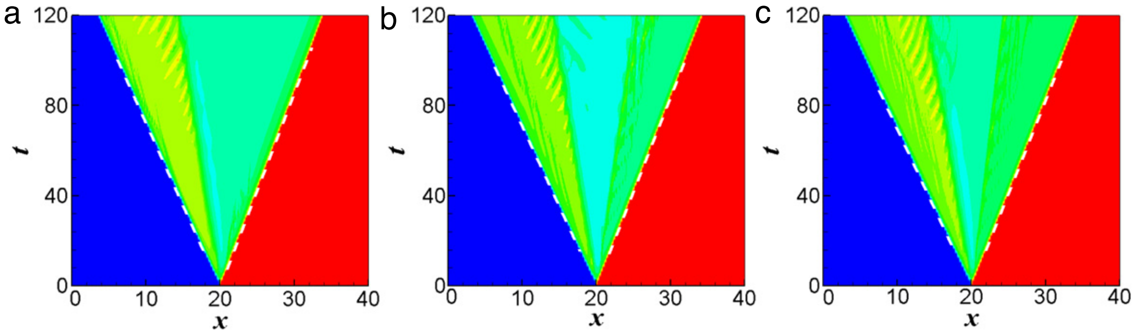

Fig. 2. (a) Spatio-temporal diagram of $\int_0^H \phi dy$ obtained using 2562×66 (a) 3842×98 (b) and 5122×130 (c) grid points. The slope of the dashed lines represent the front velocities. The rest of the parameter values are Re = 500, At = 0.05, $m = 5$, $\kappa = 0.005$ and $\theta = 60°$.