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Roll waves on a shallow layer of a dilatant fluid

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ABSTRACT

This paper is about the free surface instabilities of granular flows, usually called roll waves. A shallow layer of shear-thickening fluid ($\tau = a(\partial u/\partial y)^n$ with $n = 2$) is considered to study finite-amplitude permanent roll waves down a slope, simplified by Karman's momentum integral approach. The existence of conditions of a periodic discontinuous solution is derived, as smooth profiles with depth increasing monotonically between periodic shocks. Energy dissipation in the body of the stream and in the discontinuity is analysed and discussed. Two conditions are derived. The first is related to the physically acceptable shape of the smooth profiles, and the second is related to positive energy loss across the shock. These conditions can be converted into a limiting discharge, viewed in the fixed frame, and in a limiting flow thickness (or limiting Froude number), for the permanent periodic roll wave to exist without further conditions. A minimum-length roll wave (MLRW) is defined as the periodic permanent roll waves with zero energy dissipation in the shock. The MLRW also requires a limiting value of the Froude number to exist.

1. Introduction

Free surface instabilities of flows down inclined channels have been widely observed in Newtonian and non-Newtonian fluids. Natural gravity flows as debris flows are recorded in many areas and are a constant reminder of the need for prediction and control. In most cases, these flows manifest a succession of waves which, given enough space, develop into long waves. This behaviour is common to water and many non-Newtonian fluids. The observations of roll waves in the torrents in the late 19th century were followed by the description reported in Cornish [1]. Several authors in many areas have reported observations of roll waves in mud flows. The widespread formation of free surface instabilities, independent of the rheological properties of the fluid, is due to the effects of inertia: if the flow field could adjust its characteristics instantaneously, it would respond to a variation of the current depth reducing the local mean velocity, and the wave would simply be a kinematic wave. In real situations, the response is delayed, and an increased current depth enhances a positive mass flux, leading to the growth of the perturbation [2]. In addition, the presumed structure of the wall boundary layer favours the growth of the perturbations: the adverse gradient pressure acts to destabilize the boundary layer, reducing the wall friction and accelerating the wave crest [3]. Dressler [4] also emphasised the need for a friction reduction in the flow direction, from smaller to larger water depths, in order to obtain roll waves. Although the rheological properties of the fluid do not control the instability mechanism, all the characteristics of the development of the wave depend on those properties.

The first study of the phenomenon was based on linear-stability analysis of the basic equations written in the long-wave approximation and applied to the laminar current. It is possible to predict the threshold and growth of the waves. In most cases, laminar Newtonian flows are analysed, deriving the Orr–Sommerfeld equation for the amplitude of the perturbation [5–7]. A similar result was obtained by Chen [8], using the shallow-water equation but including the spatial variation of the momentum coefficient.

In turbulent flow in rectangular channels, assuming a Chézy resistance law with a constant coefficient, Jeffreys [9], Stoker [10] and Liggett [11] found a critical Froude number of 2. Several researchers, among them Iwasa [12], Koloseus and Davidian [13], and Berlamont and Vanderstappen [14] highlighted the strong sensitivity of the critical Froude number on the velocity profile, the Reynolds number, and friction law. In particular, according to Rouse [15] and Rosso et al. [16], the Darcy–Weisbach friction factor increases along with the Froude number in supercritical streams. According to Brock [17–19], no firm conclusion on such a dependence can be drawn, because experimental data are not accurate enough, especially the measurements of water depth. Moreover, an apparent increment in the friction factor could better be explained as an energy transfer from mean flow to waves (the limiting case is a stationary wave, with finite amount of energy and zero net flux).

Dressler [4] developed the finite-amplitude wave theory. In his paper, Dressler described that the discontinuous periodic solutions

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Nomenclature

a	Dimensional parameter in the friction law
b	Non-dimensional parameter
$b_{\text{crit}}, b'_{\text{crit}}$	First, second critical value of the non-dimensional parameter b
c, c_{crit}	Celerity of the wave, critical value of the celerity of the wave c
$F, F_{\text{crit}}, F'_{\text{crit}}$	Froude number, first/second critical Froude number
f_{crit}	Function in critical condition
g	Acceleration due to gravity
H	Height of the shock
J	Energy slope
K	Constant discharge per unit width in the moving frame
l	Length scale
m	Resistance coefficient in the Chezy formula for Newtonian fluids
n	Fluid index
q	Discharge per unit width
r	Ratio of the normal stress in the x and in the y direction
t	Time
u, U	Depth average velocity
U_b, U_f	Depth average velocity in the back/front section
U_c	Depth average velocity in the critical section
U_n	Normal velocity
v_x	Main stream fluid velocity component
x, ξ	Longitudinal coordinate
y, Y	Depth of the flow
Y_b, Y_f	Depth of the flow in the back/front section
Y_c	Flow depth in the critical section
Y_{lim}	Limiting flow depth for instability growth
Y_{max}	Maximum flow depth
Y_n	Normal depth of the flow
Y_1^*, Y_2^*	Real positive solution of the numerator of the wave profile
$\alpha, \alpha_b, \alpha_f$	Energy flux factor, in the back/front section
β, β_b, β_f	Momentum flux factor, in the back/front section
$\gamma = \rho g$	Specific weight
ΔP	Rate of change of mechanical energy in the shock
ΔE_j	Energy dissipated in the shock
ΔE_f	Mean energy dissipated for friction in a wavelength
$\lambda, \lambda_{\text{min}}$	Length of the wave, minimum value
ρ	Mass density
τ_b	Average boundary shear stress
θ	Bottom inclination
$*$	Operator indicating the non-dimensional value

are obtained by joining Bresse profiles with shocks. Dressler's theory, originally developed for fully turbulent flows, was extended to laminar flows for Newtonian fluids by Ishihara et al. [20] and to power-law fluids by Ng and Mei [21], who essentially focussed on pseudoplastic fluids (mud) and detailed the analysis only for shear thinning fluids. Prasad et al. [22] applied Dressler's theory to flowing dry grains at moderate low void concentration (the interparticle fluid is air). In their analysis, which was based on experiments, the writers assume that a large increase in the volumetric solid fraction takes place near the front of the wave. They approximate the depth-averaged dispersed flow of the grain in a manner similar to those of shallow fluid flow.

In addition, several laboratory experiments with water streams were conducted by Ishihara et al. [20], Mayer [23], Brock [18], and Julien and Hartley [24,25].

Most of the results available refer to the limit condition for the existence of roll waves, but no one can infer the determination of all roll-wave parameters (wavelength, wave height, celerity) for a given system. There are some experimental indications from Ponce and Maisner [26], who, using Brock's data [18], found that the observed periodic roll waves match the maximum growth rate (in linear-stability analysis). Referring to frequency, Kapitza [27] suggested that all waves are expected to be unstable, and the least unstable are selected by the system. He also suggested that the observed waves have the maximum absolute rate of energy dissipation. A different criterion is developed by Ng and Mei [21], who infer that the observed roll wave has the lowest amplitude corresponding to no energy loss across the shock.

Kranenburg [28] has shown that short wavelength roll waves are unstable to subharmonic disturbances, the growth of which annihilates the roll waves through the mechanism of shock coalescence and develop into roll waves of larger size. It holds true, as output of numerical integration, for small-amplitude waves, and brings to a long roll wave even though no limits to the length is given nor experimental verification is available.

We need to mention that the existence of natural roll waves (i.e. not forced at the inlet, but developed as a natural growing of instabilities) requires a minimum length of a given channel, as pointed out by Montuori [29] for turbulent conditions and by Julien and Hartley [25] for laminar conditions. Such minimum length can be extreme, making difficult the experimental observation of natural roll waves, in particular at low Froude number. Considering that the nature of roll waves in a dilatant fluid is the same than for Newtonian fluids, we can infer that a minimum length is also required for granular flows in dense regimes.

In this paper, Dressler's theory is extended to granular flows in dense regimes. The friction law for dry, granular materials flowing in a dense regime is a subject for research; our assumptions are a simplification. In the present analysis, it is assumed that grains behave like a power-law fluid, with a fluid index (the exponent of the shear rate in the constitutive equation) of $n = 2$. Dilatant fluids in which $n = 2$ were experimentally described and modelled by Bagnold [30] for granular mixture at a relatively high shear rate. They also correspond to dry, granular material at large grain-volume concentrations and moderate shear rates. We will assume this value for the fluid index, because it allows some analytical solutions which are representative of numerical solutions obtainable for a different value, but we need to mention that Chen and Ling [31,32] and Hunt et al. [33] revisited Bagnold's data and concluded that the value of the fluid index is $\sim$1.5. It is assumed that the fluid is homogeneous and that no segregation occurs.

Roll waves have been observed in experimental debris flows in a rigid bed flume with heterogeneous sediment and almost constant volume concentration [34], developing with characteristics similar to water flow roll waves. Usually most debris flows take the form of strongly transient flows, often as almost periodic surges separated by relatively low flow rate. To explain such behaviour hydraulic instability is often cited in the literature.

The necessity to include the free surface waves in modelling natural debris flows is required by a proper dimensioning of some countermeasures to reduce the risk and the damage associated with debris flows. Amongst debris flow countermeasures, direction controlling works are used to guide to safe place the stream. These works are excavated channels with cross section large enough to handle peak flow discharge levels, i.e. the surges occurring in roll waves [35].

The roll waves herein described are periodic, discontinuous solutions with a discontinuity (shock) connecting a smooth profile in a shallow-flow approximation. The waves are permanent and move downstream with a celerity higher than the maximum fluid