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Thermo-viscous fingering in quarter five-spot miscible displacements

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ABSTRACT

A pseudo-spectral method is implemented to model thermo-viscous fingering in a quarter five-spot geometry. An exponential dependence of viscosity on temperature and concentration is represented by two parameters β_T and β_C, respectively. The effects of these two parameters as well as that of the thermal lag coefficient and the Lewis number on the flow development are analyzed. Time evolutions of nonlinear fingers are examined qualitatively by plotting concentration and temperature iso-surfaces. The qualitative observations are further substantiated through an examination of the relative contributions of solutal and thermal vorticity components in the development of the instability. This study reveals that the strong velocity gradients in this radial geometry and the coupling of heat and mass transfer result in some important differences with what is obtained in the case of the rectilinear geometry.

1. Introduction

Flow displacements in porous media can result in an instability that develops at the interface between the two fluids involved in the displacement process. In the particular case where a high-viscosity fluid is displaced by a low-viscosity one, the instability manifests itself in the form of finger-shaped intrusions of the displacing fluid into the displaced one, and is commonly referred to as viscous fingering (VF). VF occurs in a wide variety of natural systems and industrial processes that include secondary and tertiary oil recovery, fixed bed regeneration in chemical processing, hydrology, soil remediation and filtration. In most applications, VF is undesirable as it results in reduced sweep efficiency of the displacement process. Any process aimed towards the elimination of the instabilities or the control of the growth rate of the viscous fingers is of high technological importance.

There is a large body of literature dealing with VF of immiscible systems in porous media [1–4]. However, since it is difficult to get detailed information about the flow development in a porous medium which by nature is opaque, most studies have focused on simpler geometries such as Hele-Shaw cells, both rectilinear and radial, as well as the five-spot geometry. Hele-Shaw cells consist of two parallel closely separated plates where a fluid is injected to displace another one that initially occupies the cell. The five-spot geometry which represents a good model of actual oil producing fields, consists of a square domain where an injection point (source) is located at the center of the square and four producing points (sinks) are located at the corners. Even though all three geometries offer a good representation of flow displacements in homogeneous porous media, there are important differences due to the geometric differences. In particular, unlike the rectilinear geometry, both radial and five-spot geometries exhibit point singularities at the sink and/or source and have a contact interface that expands radially as the flow evolves resulting in stronger velocity gradients.

The instability in Hele-Shaw cell geometries has been extensively studied in the case of *isothermal* flows. For an extensive review on such studies, see [5–7]. There is also a long history of similar studies involving *isothermal* displacements in the five-spot geometry. These studies started with the experimental works of Simmons et al. [8], Caudle and Witte [9], Habermann [10], Lacey et al. [11], Lee and Claridge [12], Demiral et al. [13] and Zhang et al. [14]. Although VF patterns observed in five-spot geometries hold qualitative similarities with those reported in rectilinear geometries [5], there are important differences resulting from flow singularities at the well locations and the stronger variations in the velocities in the quarter five-spot geometry in comparison to its rectilinear counterpart. In fact because of these last two factors, numerical modeling of the five-spot geometry is more challenging and only a limited number of nonlinear simulations are reported in the literature. These include the studies by Zhang et al. [14], Christie [15], Ewing et al. [16], Chen and Meiburg [17], Meiburg and Chen [18], Pankiewitz and Meiburg [19], Riaz and Meiburg [20], Sheorey et al. [21] and Pasarai et al. [22].

Unlike their isothermal counterparts, *non-isothermal* displacements have received relatively little attention. Among the few existing studies, one should mention the early experimental visualizations of heavy oil displacements by steam in vertical and horizontal rectilinear Hele-Shaw cells conducted by Kong et al. [23].

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Even though the study suffered from difficulties in heat transfer control and operational problems, it allowed us to reach important conclusions. In particular the authors reported that steam tended to condense in contact with the cold resident oil and that a trailing residual oil film was left coating the glass plates behind the water–oil front. A subsequent study by Sasaki et al. [24] examined drainage mechanisms at the steam chamber interface during the initial stage of a steam assisted gravity drainage (SAGD) process. It was reported that fine water droplets produced at the interface due to condensation enhanced heat transfer from the interface to the oil phase and improved the oil production. In a more relevant study, Kuang and Maxworthy [25] analyzed displacements in cylindrical capillary tubes of low temperature glycerin by the same fluid but at a higher temperature. The authors analyzed the efficiency of the displacement and identified different flow regimes that are dominated by diffusive effects, viscous effects or both. Finally, a recent experimental study by Nagatsu et al. [26] examined miscible non-isothermal displacements in a radial Hele-Shaw cell and analyzed the effects of the viscosity ratio, the injection rate and the cell gap on the finger patterns.

In terms of mathematical or numerical modeling of the *non-isothermal* case, one is forced to recognize that there is a real dearth of studies. The first serious numerical study was conducted by Saghir et al. [27] who considered nonlinear double diffusive convection in a vertically mounted homogeneous porous media. Isothermal, non-isothermal and microgravity displacements were considered. Variations of the distance traveled by the base and the tip with time were presented for each case, however only minor differences were observed between the isothermal and non-isothermal cases. A subsequent numerical simulation study by Sheorey et al. [28] analyzed both isothermal and non-isothermal *immiscible* displacements in rectangular porous formation. The authors reported that the numerical solutions experienced growth of errors during long time integration, particularly in large regions. Still, from the laterally averaged saturation profiles, the authors found that in non-isothermal displacements, the saturation profiles are front dominated and correlate well with the temperature profiles. Furthermore, Holloway and de Bruyn [29] compared experimental results with numerical simulations using FLUENT® of thermal displacements in a radial Hele-Shaw cell. Their study focused mainly on the effects of the cell gap on the instability.

There are quite a few attempts found in the literature involving linear stability analysis of the VF instability in isothermal miscible and immiscible displacements. Few recent studies have examined the stability of non-isothermal immiscible [30] and miscible [31–33] displacements. It was found that the flow instability is affected by changes associated with thermal and compositional variations. In general, if either change promotes fingering, then instability is likely to develop, although its rate of growth may be modified significantly by the coupling between the two mechanisms. More recently, Islam and Azaiez [34] carried out full nonlinear simulations of the problem in the case of a rectangular Hele-Shaw cell. Strong stabilizing effects of thermal diffusion at large *Le* flow as well as similar stabilizing influence of increased solid content at small thermal lag coefficient were observed.

The present study examines the stability of thermo-viscous flow in the five-spot geometry, which as mentioned earlier, is fundamentally different from the rectilinear geometry and more challenging to model. To the best of our knowledge, the present work constitutes the first attempt to understand the dynamics of the thermo-viscous fingering in the five-spot geometry.

The current paper is organized as follows: right after this introductory section, a mathematical model along with the physical problem will be presented. In Section 3, numerical solution of the problem is discussed. Section 4 presents the results and the conclusions are drawn in Section 5.

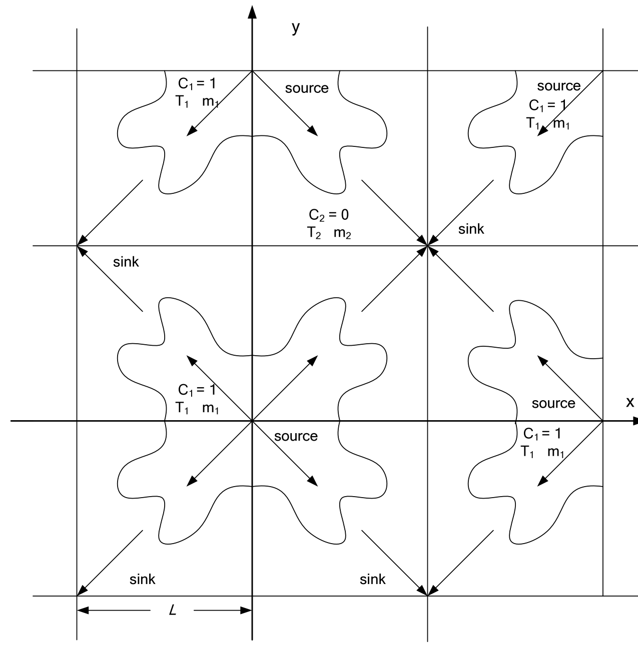

Fig. 1. The quarter five-spot configuration of injection and production wells.

2. Mathematical model

We consider a flow displacement in a five-spot geometry represented by a staggered arrangement of production and injection wells. The flow is taking place in a homogeneous medium of constant porosity ϕ and permeability k. A fluid (Phase I) of viscosity μ_1 and uniform temperature T_1 is injected at the injection wells at a rate which corresponds to the source strength per depth of $2\pi Q$. This fluid displaces the resident fluid (Phase II), of viscosity μ_2 and uniform temperature T_2, which is pushed out at the production wells (sinks). A schematic of the two-dimensional porous medium is shown in Fig. 1 where the length of each side of the quarter five-spot geometry is L. Considering that the geometry is composed of many identical building blocks, as shown in Fig. 1 and assuming that the flow is identical in each one of these, one may simply study the flow in one of such building blocks, hence the terminology *quarter five-spot*.

The flow is governed by the equations for the conservation of mass, the conservation of momentum in the form of Darcy's law and the volume-averaged mass and energy balance equations.

$$\nabla.\mathbf{u} = 0 \tag{1}$$

$$\nabla p = -\frac{\mu}{k}\mathbf{u} \tag{2}$$

$$\frac{\partial c}{\partial t} + \left(\frac{\mathbf{u}}{\phi}.\nabla\right) c = D_c \nabla^2 c \tag{3}$$

$$\frac{\partial T}{\partial t} + \lambda \left(\frac{\mathbf{u}}{\phi}.\nabla\right) T = D_T \nabla^2 T. \tag{4}$$

In the above equations $\mathbf{u}$ is the velocity vector, p is the pressure, μ is the viscosity, k is the permeability, c is the concentration of the solvent (displacing fluid), ϕ is the porosity and T is the temperature of the solid–fluid system which is assumed to be in local thermal equilibrium. The effective mass and thermal dispersion coefficients are referred to by D_C and D_T, respectively. The above equations model the propagation of two fronts, one associated with temperature and will be referred to as the thermal front while the other corresponds to mass transport and will be called the solutal front. Under the assumption of thermal equilibrium between the solid and fluid, *the thermal front travels slower than the fluid front, as heat*