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Vortex formation in lid-driven arc-shape cavity flows at high Reynolds numbers

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Abstract

High-Reynolds number lid-driven flow in arc-shape cavities with different cross sections is considered up to $Re = 8000$. The unsteady streamfunction–vorticity transport formulation is adopted and a second order finite difference numerical method is applied to computational grids generated by body-fitted coordinate transformation. The effects of aspect or arc angle ratio r, on the formation and growth of vortical structures, as well as on the existence and development of periodic solutions are discussed. It is found that for the case where $r > 1/2$, only a secondary vortex appears in addition to a primary core vortex, for stationary solutions, whereas tertiary and quaternary vortices appear for the cases where $r < 1/2$, near the curved wall. Periodic solutions at high Reynolds numbers are observed when $r \geqslant 1/2$; while transient oscillations decay in time for $r < 1/2$.

Keywords: Lid-driven flow; Arc-shape cavity; Vortex formation; High Reynolds number

1. Introduction

The flow problem in lid-driven cavities has been a popular research subject due to its wide range of physical applications for which the structure of inertia induced vortices in a cavity needs to be investigated in detail. The lid-driven cavity problem has also been used as a classical benchmark case to test new numerical schemes and methods [1–3]. Most of the studies in the literature are concerned with the square or rectangular cavity flows, although in applications, the cavities may be non-rectangular. There are few studies dealing with flows in curved cavities driven by a moving lid. Tillmark [4] carried out an experimental and numerical study on the lid-driven flow in a polar cavity. In a series of papers, Cheng et al. [5–7] studied experimentally and numerically the effects of buoyancy and convective heat transfer on the flow pattern inside an arc-shape cavity. Migeon et al. [8] studied experimentally the effects of lid-driven cavity shape on the flow establishment phase for square, rectangular and semi-circular cavities. Recently, Glowinski et al. [9] applied a finite element method to the wall-driven flow in a semi-circular cavity and revealed the vortex structure at high Reynolds numbers. They also identified a Hopf bifurcation leading to periodicity of solutions around a Reynolds number of $Re = 6600$.

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In the present work, we aim to study the formation and establishment of vortical structures at high Reynolds numbers up to $Re = 8000$, in arc-shape cavities of different cross sections, including the semi-circular cavity as a test case. We also aim to observe periodicity of solutions in transient and steady-state flow regimes, and to identify the effect of cross sectional shape determined by the aspect ratio or arc angle ratio r, on the existence of periodic solutions, reported in [9] for a semi-circular cavity. The unsteady streamfunction-vorticity transport formulation is adopted. Body fitted coordinate transformation is applied to generate an elliptic computational grid. The governing equations are discretized in space using second-order finite differences. The streamfunction equation is solved using the iterative method of successive over relaxation with Chebyshev acceleration. Coupled to it, the vorticity transport equation is solved in time using a second-order explicit Adams–Bashforth scheme. The primary, secondary and higher vortical structures formation and establishment are compared for different arc-shape cavity cross sections. The existence of periodic solutions in transient and steady-state flow regimes is investigated.

2. Formulation and numerical solution method

We consider an arc-shape cavity of height H and with a lid of length L at the top, moving at a constant speed U_0 to the right. Different cross sectional cavities are considered, which correspond to different aspect ratios H/L. They are also characterized by the ratio, r, of the arc angle to 2π. Five different cross sections are considered, corresponding to $r = 2/3$, $r = 1/2$, $r = 1/3$, $r = 1/4$ and $r = 1/5$ ratios. The arc angle ratio $r = 1/2$ corresponds to semi-circular case.

It is possible to transform the physical domain of the cavity to a rectangular computational domain in two dimensions. The transformation gives rise to a body fitted coordinate system [10,11], in which the coordinate lines are given by the images of uniform coordinate lines in the computational domain. We then consider the mapping from the computational domain (ξ, η) to physical domain (x, y). The elliptic grid generation equations are given as,

$$\alpha x_{\xi\xi} - 2\beta x_{\xi\eta} + \gamma x_{\eta\eta} = 0, \tag{1}$$

$$\alpha y_{\xi\xi} - 2\beta y_{\xi\eta} + \gamma y_{\eta\eta} = 0 \tag{2}$$

where the subscripts denote partial differentiation and α, β, γ are transformation metrics which are written as,

$$\alpha = x_\eta^2 + y_\eta^2, \tag{3}$$

$$\beta = x_\xi x_\eta + y_\xi y_\eta, \tag{4}$$

$$\gamma = x_\xi^2 + y_\xi^2. \tag{5}$$

The grid orthogonality is controlled by vanishing second transformation metric β. In this work, for the grids that have been generated by the elliptic grid generation technique, the order of magnitude of the second transformation metric β is 10^{-6}. An example of a generated elliptic grid is given in Fig. 1.

To model the flow inside the cavity, we use streamfunction-vorticity transport formulation. Choosing the lid length L as a length scale, and the lid speed U_0 as a velocity scale, the non-dimensional flow equations in curvilinear coordinates are,

$$\alpha\psi_{\xi\xi} - 2\beta\psi_{\xi\eta} + \gamma\psi_{\eta\eta} = -J^2\omega, \tag{6}$$

$$\omega_t + \frac{1}{J}\big[(y_\eta u\omega_\xi - y_\xi u\omega_\eta) + (x_\xi v\omega_\eta - x_\eta v\omega_\xi)\big] = \frac{1}{Re\,J^2}(\alpha\omega_{\xi\xi} - 2\beta\omega_{\xi\eta} + \gamma\omega_{\eta\eta}) \tag{7}$$

where ψ represents the streamfunction, ω is the vorticity, u and v are the velocity components in (x, y) plane and J is the Jacobian of the transformation which is written as,

$$J = x_\xi y_\eta - x_\eta y_\xi. \tag{8}$$

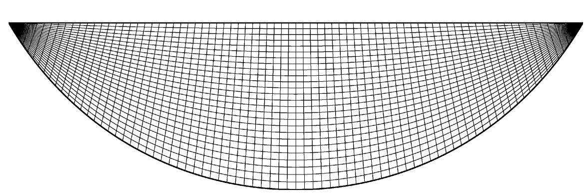

Fig. 1. (27 × 81) grid sample for $r = 1/3$ arc cavity.