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# Interactive boundary layer models for channel flow

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**Abstract**

In this paper, the aim is to present the results of a new approach for the asymptotic modeling of two-dimensional steady, incompressible, laminar flows in a channel. More precisely, for high Reynolds numbers, the walls of the channel are deformed in such a way that separation is possible. Of course, numerical solutions of Navier–Stokes equations can be calculated but it is believed that an asymptotic analysis helps in the understanding of the flow structure. Numerical solutions of Navier–Stokes equations are compared with solutions of asymptotic models included in a more general model called global interactive boundary layer.


*Keywords:* Interactive boundary layer; Separation; Channel flow; Asymptotic analysis

## 1. Introduction

In the study of high Reynolds number flows past a solid wall [1,2], Navier–Stokes equations reduce formally to Euler equations far from the walls. Near the walls, the corresponding inviscid approximation is corrected by a boundary layer solution which takes into account viscosity effects. The standard approach is hierarchical in the sense that the inviscid solution is calculated independently from the viscous solution and then the boundary layer effects are introduced.

Here, we consider a steady, two-dimensional, incompressible, laminar flow in a channel with small wall perturbations. These perturbations, such as indentations, are sufficiently significant to produce streamwise adverse pressure gradients which cannot be neglected. Physical phenomena such as flow separation or upstream influence cannot be described by following the standard approach.

The hierarchy introduced in the standard theory and according to which the knowledge of the inviscid solution enables us to calculate the boundary layer is broken. A strong coupling between the viscous and inviscid zones takes place.

Then, the alternative is either to solve the complete Navier–Stokes equations or to find simpler models in which the boundary layer and Euler equations are no longer hierarchized. These models form the Interactive Boundary Layer

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theory, IBL. The analysis of IBL models leads to a better understanding of the flow structure and of the corresponding significant scales.

The mathematical justification of IBL is the asymptotic analysis but we are faced to a paradox since, formally, when the study is carried out for a Reynolds number tending to infinity, the hierarchy imposes itself in a natural way. Essentially, this is the result of the construction process of the asymptotic expansions which leads to expansions of Poincaré type called regular expansions. With regular expansions, the paradox is apparently broken if it is possible to match directly the component of velocity normal to the wall between the viscous and "inviscid" zones. In external flows, the triple deck theory answers these questions [8–10,17]. For channel flows, the counterpart of the triple deck theory has been established by Smith [13,14]. However, these theories introduced very restricting hypotheses on the scales of the flow.

In order to overcome these limitations, it is much better to use generalized expansions which are such that a small parameter of the problem can be present in the functions constituting the expansions. Then, the inverse of the Reynolds number and the thickness $\varepsilon$ of the boundary layer may not be linked so tightly and may not be so small as in Smith's theory. In this way, we can take into account effects which are no longer considered as being of second order.

Two methods can be used. In the first one, different domains are considered. In one domain, the Euler equations are valid and in the other ones the boundary layer equations apply. Obviously, the Eulerian zone is not exactly inviscid because, in the case of strong coupling, the effects of viscosity must be represented which is not possible with regular expansions. These methods are generalizations of the method of asymptotic expansions with a matching principle called modified Van Dyke's principle [2].

In the second method, the flow is analyzed with the Successive Complementary Expansion Method, SCEM, in which we seek a uniformly valid approximation in the whole domain. Thanks to generalized expansions, the effect of the boundary layer on the Eulerian region and the reciprocal effect are considered simultaneously. This method, developed by Cousteix and Mauss [1,2] is based upon the idea that the reasoning used in the method of matched asymptotic expansions must be inverted. A structure of a uniformly valid expansion is assumed and then the method of construction is deduced. If we know that the flow is well approximated by a solution of Euler equations in a certain domain, the question is to determine what must be added to obtain a uniformly valid approximation of the solution of Navier–Stokes equations. This method has been used by Dechaume et al. [4] to calculate channel flows with small wall deformations.

More precisely, in a channel, there is no external flow region and the asymptotic models for the flow perturbations are mainly based on an inviscid rotational core flow region together with boundary layers near the walls. The asymptotic analysis of these flows has been performed essentially by Smith [13–15] and a systematic approach has been proposed later by Saintlos and Mauss [12]. A comprehensive discussion of this structure can be found in Sobey [16]. The modelling of channel flow has also been examined by Lagrée et al. [5] and by Lagrée and Lorthois [6] who consider in particular the flow in axisymmetric pipes.

In the present paper, the formulation of the problem includes two parameters, the Reynolds number $\mathcal{R}$ and the thickness $\varepsilon$ of the boundary layer generated by the walls indentations (Section 2). A consistent model is obtained by coupling Euler equations, valid in the flow core for large Reynolds numbers, and Prandtl equations, valid in the whole field for small values of $\varepsilon$ (Sections 3–5). The interaction between the two sets of equations is primarily based on the pressure. In Section 6, a simplified method for the pressure calculation is obtained for a class of indentations which contains the critical case of Smith's theory. Results are compared with those obtained from a numerical solution of Navier–Stokes equations. Another method for calculating the pressure is based on the linearized form of a more complete model. The formulation of the model, its numerical solution and results are given in Section 7.

## 2. Formulation of the problem

Dimensionless Navier–Stokes equations can be written

$$\mathbf{div}\,\vec{V} = 0, \quad (\mathbf{grad}\,\vec{V})\cdot\vec{V} = -\,\mathbf{grad}\,\Pi + \frac{1}{\mathcal{R}}\Delta\vec{V} \tag{1}$$