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Weakly nonlinear EHD stability of slightly viscous jet

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ABSTRACT

A weakly nonlinear approach is utilized here to study the electrohydrodynamic (EHD) instability of an incompressible viscous liquid jet stressed by an axial electric field. The linear motion equations is solved in the light of nonlinear boundary conditions. The viscosity is assumed to be small. The study takes into account both the shear and radial components of the stresses at the interface. In the linear theory, we discuss the breakup phenomena of liquid jets. Also, it is found that, the electrical shearing stresses have no effect at the linear marginal state, while the linear cutoff wavenumber depends on the electrical shearing stresses. A nonlinear perturbation method is introduced. This method can be described our problem precisely. The nonlinear stability is compared with the linear stability condition in the weak viscosity case. It is found that, the weak viscosity has effect on the nonlinear stability condition, in contrast with the linear analysis, whereas the nonlinear cutoff wavenumber doesn't depend on the weak viscosity in both the linear and nonlinear theory.

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1. Introduction

Electrohydrodynamic (EHD) is the study of fluid motions driven by external electrostatic fields. The process of EHD is dependent on so many parameters and physical properties of the fluid system. There has been continued interest in the behavior of liquid jets under the influence of an electric, because of the numerous industrial applications, such as in paint spraying [1], electronic ink-jet printers [2], etc. The study of capillary liquid jet instability using hydrostatic theory, first explained by Plateau [3]. He showed that the axisymmetric deformation is stable or unstable according as the wavelength of deformation of the cylindrical surface is less than or greater than the circumference of the cylinder. Rayleigh [4] extended Plateau's work using hydrodynamic theory of linear stability. He developed the important concept of the mode of maximum instability by treating liquids as perfect conductors. Most studies have tended to consider either perfect conductors [3,4] or perfect dielectrics [5]. Taylor [6] proposed an EHD theory based on the leaky dielectric model. This model accounts for the charge accumulation at the interface due to finite conductivities in the fluids, where the surface tangential electric stresses induce fluid motion. In the light of linear theory of leaky dielectric theory Melcher and Taylor [7] explained certain paradoxical phenomena pertaining to nonconducting fluids. Saville [8] examined the linear EHD stability of an infinite fluid cylinder in the presence of an axial electric field. Both fluids were treated as leaky dielectrics. He showed that a leaky dielectric requires much lower field strength than a perfect dielectric for jet stabilization to take place. In addition he showed that the stability of the cylindrical configuration depends on the relative magnitude of the conductivity and dielectric constant ra-

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tios. Experiment have been reported by Sankaran and Saville [9] on leaky dielectric bridge. Their results showed that, the stability with the application of an electric field depending on the conductivities and dielectric constants of the two media as predicted by the leaky dielectric model [8]. Burcham and Saville [10] studied the stability of a leaky dielectric bridge penned between planar electrodes held at different potentials and surrounded by a nonconducting, dielectric gas. The stability conditions of the perturbed system are discussed both theoretically and numerically. Burcham and Saville [11] compared the theoretical results with experimental work, that demonstrated how an electric field stabilizes an otherwise unstable configuration. Pelekasis et al. [12] studied the linear oscillations of viscous, capillary bridge in the presence of an axial electric field. They obtained, the stability conditions for both cases of leaky and perfect dielectrics. López-Herrera et al. [13] studied the linear electric viscous jets. They discussed the role of limited conductivity and permittivity on the behavior of electrified jets for viscosity limit, low and high electrical conductivity and permittivity. Elcoot [14] investigated the effect of a uniform surface charge in the presence of a finite rate of charge relaxation of cylindrical interface. He examined the effects of the surface charge and charge relaxation on the stability of the flow by considering various limiting cases in axisymmetric and nonaxisymmetric modes. He predicted a new unstable regions.

The nonlinear problem of the leaky dielectric model has attracted the attention of many investigators. By treating nonlinear processes rigorously, Feng and Scott [15] improved agreement between the theory and experiment for higher field strengths and larger deformations. Feng [16] extended the computations of Feng and Scott [15] to include the charge convection effect that is expected to emerge when the flow intensity is considerable. Theoretical treatments of the nonlinear aspects of the effect of an axial electric field on the streaming instability of surface waves, which propagating through porous media of a cylindrical flow of two concentric finitely conducting fluids, have been investigated by Elcoot and Moatimid [17]. They showed that the nonlinear theory predicted more accurately the instability, where new instability regions, appeared due the nonlinear effects. The nonlinear electroviscous potential flow analysis has been studied by Elcoot [18]. He showed that, the nonlinear stability condition depend on the viscosity coefficients, which does not explain in the linear theory of viscous potential flow analysis model of Funada and Joseph [19,20]. In their model, they considered the normal stress is not neglected, but the effect of shear stress is neglected. Elcoot [21] introduced new technique based on the perturbation theory. He derived a new condition on the material properties, involving weak electric relaxation times in both fluids. Such effects can only be understood by nonlinear analysis, as the linear analysis fails to predict them.

A generalization of the nonlinear instability for viscous flow is a very difficult problem. The difficulty arises as the nonlinear terms are considerable. Fing and Bear [22] restrict themselves to the case of weak viscous effects. This weakness is regarded such that viscous effects appear at the interface and gradually decrease to be neglected in the bulk [23–25]. Their treatment based on the viscous or viscoelastic contribution has been demonstrated through the normal stress boundary condition. While, the tangential stress is ignored. In this paper we employ the nonlinear analysis based on the perturbation technique [21] to describe the stability of jet in the small viscosity case, under the influence of an axial uniform electric field. The study takes into account the shear and normal stresses effects at the interface.

2. Governing equations

We are interested in examining the stability of an infinite incompressible cylindrical jet of radius R, under the influence of an axial uniform electric field E_0. In what follows, the subscripts 1 and 2 denotes variables associated with the fluids inside and outside the jet, respectively. Bulk properties of the liquid (density ρ, viscosity μ, dielectric constant ε_1 and electric conductivity σ_1) as well as interfacial properties (surface tension T) are uniform and constant under the isothermal analysis. In the most practical applications the surrounding material is a gas and, thus it is assumed that it has negligible density and viscosity, but uniform and finite dielectric constant ε_2 and electric conductivity σ_2. The gravitational acceleration is ignored. The motion ensues from rest and the flow field generated due to wave motion. To describe the fluid motion, we use the moving frame of reference with the jet at rest. If (r, Z, t_0) is the coordinate system for the traveling jet and (r, z, t) for the jet rest, the transformation connecting the two systems is given by $z = Z - u_0 t_0, t = Z/u_0$ where u_0 is the uniform speed of jet along the axis of the cylinder, as [5,14]. For convenience, the usual cylindrical coordinates (r, θ, z) is used. Only the axisymmetric case is considered in this study. The interface between the liquid and gas is assumed to be well defined and initially cylindrical. The Maxwell equations lead to an exponential decay of the bulk charge density as $\exp(-\sigma_1 t/\varepsilon_1)$ where the parameter σ_1/ε_1 is sufficiently short, so that the electric charge density in the bulk is essentially zero. Therefore, the bulk forces of electrical origin are negligible and the field coupling occurs at the interface as specified by the appropriate boundary conditions [7].

The idea for the weakly nonlinear description is the some slight departure from the linear viewpoint [24,25]. At this end, the nonlinear problem will contain the linear description with some additional terms representing a correction for the main solution. The weakly nonlinear description given here depends on neglecting the nonlinear terms from equations of motion and applying the appropriate boundary conditions without dropping the nonlinear terms. At this stage, the dispersion relation should be extended to include nonlinear terms.

The electric potential Φ governed by Laplace equations

$$\nabla^2 \Phi_1 = 0, \tag{2.1}$$

$$\nabla^2 \Phi_2 = 0, \tag{2.2}$$

where ∇^2 is axisymmetric cylindrical Laplacian operator defined as

$$\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{\partial^2}{\partial z^2}. \tag{2.3}$$

The Laplace equations (2.1) and (2.2) satisfied the requirement of steady-static charge conservation in the bulk fluid, as expressed in terms of zero divergence of electric current density due to Ohmic conduction.

The conservation equations of mass and momentum for the liquid jet are

$$\nabla \cdot \mathbf{v} = 0, \tag{2.4}$$

$$\rho\left[\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{v}\right] = -\nabla \Pi + \mu \nabla^2 \mathbf{v}, \tag{2.5}$$

where $\mathbf{v}$ is the fluid velocity vector and Π is the modified pressure defined by

$$\Pi = p - \frac{1}{2}\varepsilon_j E_0^2, \quad j = 1, 2, \tag{2.6}$$

and p is the hydrostatic pressure. The relaxation time is much smaller than the liquid oscillation time. This prevents any free charge from appearing in the bulk of the liquid, and thus no electric stresses arise in Eq. (2.5).

We consider a perturbation, so that the surface deflection $S(r, z, t)$ is expressed by

$$S(r, z, t) = r - R - \eta = 0, \tag{2.7}$$