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Analytical solutions for a single blade in vertical axis turbine motion in two-dimensions

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ABSTRACT

An analytical model for a time dependent two dimensional flow around a moving profile is developed. The model is suitable for fast aerodynamic and aeroelastic coupling calculations. It determines the inviscid pressure distribution in the vicinity of one blade and the force on the blade in arbitrary two dimensional motion. The method is more flexible than previous analysis: it can represent any profile, pitching motion and blade attachment position. The method is based on conformal mapping techniques and Laurent's series decomposition and is faster and more accurate than standard panel methods. A main idea is to directly treat the singularities of the flow in a mapped plane where any geometrical plane is simplified to a circle. The vorticity is assumed to be shed in the form of a continuous vortex sheet near the trailing edge.

1. Introduction

The turbine in the present investigation has a vertical shaft connected to straight vertical blades via support beams instead of the horizontal shaft used in conventional wind turbines (see Fig. 1).

The vertical axis wind turbine concept was invented by J.M. Darrieus [1] in 1931, and a renewed interest for the concept appeared during the oil crisis in the 1970's.

The vertical axis wind turbines (or VAWT) are again considered for large and small scale wind power generation, see Fig. 1. They can also be used to produce energy from underwater currents. At Uppsala University, the main focus is on simple turbine construction to minimize costs [2]. The effort includes the usage of improved composite materials, wind turbine aerodynamics and new generations of permanent magnet high voltage generators that avoid mechanical gears. However, in particular two key issues need to be resolved.

Firstly, the vertical axis turbines experience a complicated flow with remaining theoretical uncertainties. An efficient tool had not been previously developed for a systematic theoretical design study of the pitch angle, attachment point and shape of the blades. In most experimental and theoretical studies the old NACA 4 digit series airfoil has been used [3].

Secondly, the structural design of the turbine is demanding. Hence, without the use of special composites materials and accurate models it is difficult to build a reliable turbine [4]. This could be a major reason why the worldwide research efforts on vertical axis turbines have declined for the last two decades. Most of the previous vertical axis program stopped due to failures of a blade or the main bearing. After the damage of the world's largest vertical axis turbine in the 1990's, the 3.2 MW rated power Eole in Cap Chat at Quebec [4], almost all large scale Darrieus and H-rotor research projects have been stopped.

It is commonly believed that vertical turbines are much less efficient than their horizontal counter parts. However, it should be kept in mind that commercial horizontal wind turbines have been optimized for more than twenty years. For instance, the reported maximum measured power coefficient [5] of the VAWT 260 in Great Britain is as high as 0.39, which twenty years ago was an acceptable figure of merit. Substantial progress could be expected if the understanding of the physics of vertical turbines would be deepened.

The design of the turbine can be improved, for instance by minimizing the drag produced by struts and junctions. Their special aerodynamic features such as vortex shedding may turn out to be advantageous for small and medium size electricity production from wind or underwater streams (vortex shedding is used by insects and fish to produce a very powerful thrust [6]). Finally, the overall design could be simplified and thus be made more cost efficient.

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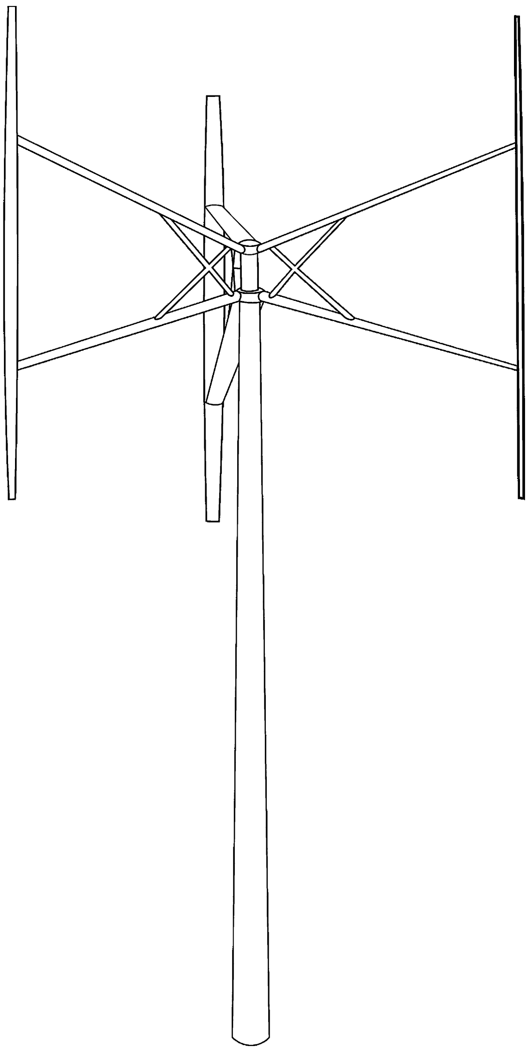

Fig. 1. General view of an H-rotor with three blades.

2. Previous aerodynamic models

The methodology described in this paper has been developed with emphasis on two peculiarities of cross flow turbines: the complicated flow surrounding vertical turbines and the sensitive dependency on various aerodynamic parameters. Features that need to be investigated in more detail are: the unsteady interaction between the blades due to the continuously changing angle of attack via vortex shedding and unsteady relative flow curvature seen by the blades during rotation [7]. Additionally, the wing profile affects strongly the dynamic stall performance. Some of these effects may be studied more efficiently with the method described here.

Previously, vertical axis turbines have been modeled in three ways: (i) a streamtube analysis (also called blade element momentum theory) [3,8] originating from the horizontal axis turbines theory, (ii) vortex models [9–12] and (iii) Computational Fluid Dynamics (CFD) models [13–15]. The third group of models may be more accurate and reliable, but is more demanding in terms of computing effort and time as compared to the others. The two first models typically need inputs in the form of lift and drag data as determined from computations or experiments. In most cases this set of data is taken from non moving wind tunnel experiments or steady state calculations. As the local flow conditions are radically different, the results from such static tests cannot be considered. Moreover, the static data needs to be related to the fully interactive flow encountered by the wings. This is usually done by calculating the relative angle of attack seen by the blade in motion. However, even the definition of this effective angle of attack may be questionable in view of the complicated flow experienced by real vertical turbine blades. A simple analytical model is presented in this paper to clear these uncertainties concerning two of the aforementioned aspects: flow curvature influence and blade–wake interaction.

The present work has been inspired by three previous studies. One of the most complete models for vertical axis turbines is a vortex model developed by Oler and Strickland [9], but it disregards flow curvature. The use of the panel method complicates the vortex shedding treatment and increases the computational cost. The panel method, however, can be generalized to three dimensions unlike methods based on conformal mapping.

The analytic study of Wilson [10] uses the conformal mapping from a flat plate to a circle via the Joukowsky transform. The advantage of his study is the analytical simplicity. However, the flat plate is not a suitable airfoil for vertical axis turbines (it has not a preferential direction of rotation) and generalizations are necessary. Another study was carried out by Zervos et al. [16] using the results of Couchet [17] for the aerodynamics of arbitrary two dimensional profiles in general two dimensional motion. It was applied to a NACA0012 airfoil but the oversimplified computation [16] did not give convincing results.

Zannetti [18] recently applied a discrete vortex model to study vortex trapping blades for vertical axis turbines. However, the forces on the blades were not evaluated. Moreover it does not seem straightforward to apply that model to N bladed turbines or N boundary problems. The computation time in the present paper is reduced by using a suitable implementation of the Fast Multipole Method (FMM) algorithm [19]. Wang [12], also recently applied a 2D panel-method to the simulation of the flow around a multi-bladed conventional VAWT.

The model presented in this paper is a generalization of Wilson's [10] vortex model to an arbitrary profile in arbitrary motion. Therefore the model will inherit the flexibility of Oler's [9] model and benefit from exact calculations to decrease significantly the computing time. The generalization to the N bladed case has been investigated by Österberg [20]. The present model is able to determine the inviscid pressure distribution on the rotating blade. Some possible future developments of this analysis (influence of viscosity and aeroelastic computations) are not developed here but briefly given in Section 8 as perspectives.

The model is presented in several steps. Firstly, the geometry of the physical problem is defined and simplified via conformal mapping techniques. The set of equations to be solved together with the appropriate boundary conditions are established and solution details are given. Exact formulas for the forces from the fluid on the blades are also derived. Explicit results are presented for an application to vertical axis turbine flows.

3. Geometry definition, Laurent's series and conformal maps

A conformal mapping is a transformation which preserves local angles in the transformation. Riemann [21] showed that there exists a conformal map which transforms the exterior of any shape (here an airfoil) onto the exterior of the unit disk. The conformal mapping transformation is analytic. It follows that there exists a Laurent series expansion of this transform which enables a fast evaluation of the mapping [22] and simplifies the treatment of derivatives and integrations over the airfoil contour.

The airfoil coordinates are given by a set of discrete coordinates (x_{Ci}, y_{Ci}) converted into a set of complex numbers $z_{Ci} = x_{Ci} + iy_{Ci}$.