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On the role of unsteady forcing of tracer gradient in local stirring

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ABSTRACT

Local stirring properties in two basic mixing flows – namely, the blinking vortex and the sine flow – are studied through the tracer gradient approach. The velocity gradient tensor and related quantities such as the strain persistence parameter are derived from the analytical velocity fields. Numerical Lagrangian tracking of the gradient of a tracer shows how local stirring is affected by forcing experienced through strain persistence. In both flows Lagrangian variations of strain persistence occuring on a time scale shorter than the response time scale of the tracer gradient lead the latter to align close to the direction determined by the mean strain persistence. It is the special alternating behaviour of strain persistence resulting from flow operation that makes this direction coincide with the local compressional strain direction for both the sine flow and the clockwise/counterclockwise blinking vortex. The rise of the tracer gradient and thus local stirring are in turn promoted by this statistical alignment.

1. Introduction

Mixing phenomena in fluid flows are far from being fully explained, despite significant progress in the study of scalar transport by laminar or turbulent flows. In a number of cases questions such as "which stirring gives the most efficient mixing?" or "how long does it take to reach a given mixed state?" remain unanswered. Actually, there is still an intense need of basic studies devoted to mixing phenomena, more especially as many industrial processes use mixing flows. In chemical processes, for instance, reactions may take place in a poorly mixed medium, well before homogenization is achieved, thus causing damage of product quality or bringing about an excess of pollution. Developing processes which ensure energy sparing, improvement of safety and productive capacity together with low pollutant release therefore needs designing devices in which mixing would be well understood and controlled.

As is well established, mixing in fluid flows is essentially a matter of stirring and molecular diffusion. Stirring makes fluid-element pairs separate which is also seen as stretching of material lines or surfaces. It is taken for granted that stretching promotes mixing. Indeed the higher the stretching, the larger the contact area and the smaller the distance between fluid portions to be mixed. Hence the hastening of molecular diffusion and homogenization of the mixture. In addition, "good mixing" often implies a uniform distribution of stretching rate within the flow.

Understanding the advective part of mixing during which the mechanical action of fluid flow produces small scales of the scalar field – heat, contaminant ...– through "cascade phenomena" without significant influence of molecular diffusion is a big issue of turbulent mixing, especially in large-Péclet-number flows [1]. The passive scalar cascade and the way in which it may be connected to mixing studies have been investigated through the statistical approach [2,3]. The study of advective mechanisms of mixing is also relevant to chaotic mixing [4,5]. In this regard, different approaches have been used, namely those based on Lyapunov exponents [4], on the concept of effective diffusivity [6] or on geometrical properties of flows defined through stable and unstable manifolds [4]. The latter geometrical approach, in particular, provides a thorough knowledge of mixing properties of two- and three-dimensional unsteady flows. Efficiency of mixing, for instance, may be qualitatively diagnosed through the detection of barriers to transport.

The approach based on the evolution of the gradient of a passive scalar – or tracer – transported by the flow arises from a natural idea, for cascade mechanisms result in production of large gradients and micromixing efficiency is measured by the mean dissipation rate of tracer fluctuations, a quantity proportional to the variance of the tracer gradient [3]. The rise of the local, instantaneous gradient of a non-diffusive tracer is equivalent to the stretching of fluid elements and both mechanisms have been used to investigate turbulent [7,8] as well as chaotic mixing [9,10]. It is

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also worth mentioning that properties of Lyapunov vectors and exponents can be understood through the tracer gradient dynamics [11]. In addition, by providing quantitative insight into local stirring properties, the tracer gradient approach is to a great extent complementary to the geometrical approach. The tracer gradient properties are even basically connected to the structure of mixing patterns in that the gradient direction and magnitude respectively correspond to the striation orientation and thickness – fine structures meaning large gradients –. Moreover, as concentration contours tend to align with manifolds in chaotic regions and almost homogeneous tracer patches lie in islands, mixing patterns are tightly linked to coherent structures in tracer trajectories.

Production of the norm of a tracer gradient – which in turn promotes mixing through the resulting accelerated molecular diffusion – rests on both strain intensity and gradient orientation with respect to strain principal axes. Orientation of tracer gradient in the strain basis stems from the combination of strain, vorticity, strain basis rotation and molecular diffusion [12]; strain basis rotation itself depends on strain, vorticity, pressure and viscous effects [12] and so do the strain eigenvalues. These established facts suggest the intricacy of the mechanisms underlying the mixing process. In three-dimensional flows analytical approaches are unfeasible unless simplifying hypotheses on the dynamic field are made [13]. Assuming the flow to be two-dimensional and neglecting molecular diffusion make the problem somewhat simpler and even analytically tractable in some special cases [7].

Even so, mixing problems have most often to be addressed in unsteady conditions. Actually, non-stationary aspects of the mixing process are essential in the turbulent as well as in the chaotic regimes. In the view of the tracer gradient approach, then, one has to examine mixing properties of fluid flows through the dynamics of the tracer gradient, namely through its response to varying mechanical actions or forcing. The dynamics of scalar gradient alignment, for instance, has been addressed in some previous studies [12,14,15]. Recently, it has also been shown that in two-dimensional flow – more specifically in the laminar Bénard – von Kármán street [16,17] – forcing through Lagrangian variations of strain persistence may deeply affect tracer gradient behaviour in terms of alignment properties and norm growth rate. In particular, local orientation of the tracer gradient appears to be strongly dependent on whether or not the gradient responds to this kind of forcing. Further investigation needs addressing some remaining questions, more especially: i) is the latter behaviour observed in various flows or is it restricted to some class of flows? and ii) how are local stirring properties of the flow determined by this tracer gradient dynamics?

The present study deals with these questions by analysing the kinematics of the tracer gradient in two model mixing flows. We more specifically focus on the conditions in which local stirring may be enhanced by unsteady forcing. This is done in Section 3 in which the tracer gradient behaviour is investigated numerically starting from the analytical definition of the flow. Theoretical bases on the dynamics of the gradient of a non-diffusive tracer in two-dimensional flows are given in Section 2. Section 4 is devoted to conclusion.

2. Stirring properties considered through the behaviour of the tracer gradient

We address the mechanisms of local stirring, that is, the advective part of mixing during which fine structures form in the mixing pattern. Since small-scale structures such as thin filaments imply high gradients, this process also finds expression in enhancement of the gradient of a non-diffusive scalar or tracer convected by the flow. The analysis is relevant to the mixing of high-Schmidt number quantities as well as to the large-Péclet-number regime in which a significant stage of the mixing process is filled by production of small scales by stretching before diffusive homogenization takes place.

In two-dimensional flows, then, the behaviour of the tracer gradient can be rigorously analysed from the velocity gradient properties [7,15]. In this approach the tracer gradient, $\boldsymbol{G}$, is determined through its orientation and norm which are given by Eqs. (1) and (2) [7]:

$$\frac{\mathrm{d}\zeta}{\mathrm{d}t} = \sigma(r - \cos\zeta), \tag{1}$$

$$\frac{2}{|\boldsymbol{G}|}\frac{\mathrm{d}|\boldsymbol{G}|}{\mathrm{d}t} = -\sigma\sin\zeta, \tag{2}$$

in which $\boldsymbol{G} = |\boldsymbol{G}|(\cos\theta, \sin\theta)$ and $\zeta = 2(\theta + \Phi)$ gives the gradient orientation in the local strain basis (Fig. 1). Orientation of strain principal axes, Φ, is defined by $\tan(2\Phi) = \sigma_n/\sigma_s$ where $\sigma_n = \partial u/\partial x - \partial v/\partial y$ and $\sigma_s = \partial v/\partial x + \partial u/\partial y$ respectively denote the normal and shear components of strain; u and v are the velocity components. Strain rate, σ, is given by $\sigma = (\sigma_n^2 + \sigma_s^2)^{1/2}$. In Eq. (1) r is the strain persistence parameter which measures the respective effects of effective rotation – vorticity plus rotation rate of strain principal axes – and strain and is defined as:

$$r = \frac{\omega + 2\mathrm{d}\Phi/\mathrm{d}t}{\sigma}, \tag{3}$$

where $\omega = \partial v/\partial x - \partial u/\partial y$ is vorticity.

As shown by Lapeyre et al. [7], the strain persistence parameter, r, defines a criterion to partition the flow in regions with different stirring properties. In strain-dominated regions, $r^2 < 1$, the tracer

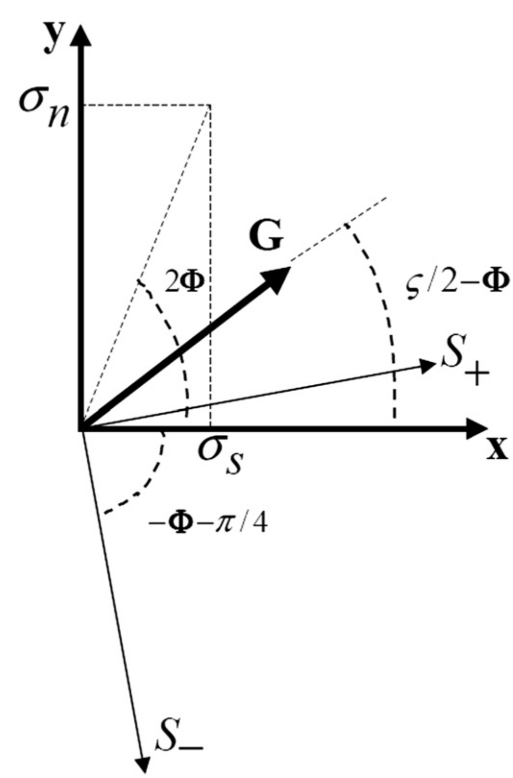

Fig. 1. Schematic of reference frames; S_- and S_+ respectively stand for compressional and extensional strain axes.