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Injection and coalescence of bubbles in a quiescent inviscid liquid ☆

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Abstract

Time periodic generation and coalescence of bubbles by injection of a gas at a constant flow rate through an orifice at the bottom of a quiescent inviscid liquid is investigated numerically using a potential flow formulation. The volume of the bubbles is determined for different values of a Weber number and a Bond number. Single bubbling and different regimes of coalescence are described by these computations. The numerical results show qualitative agreement with well-known experimental results for liquids of low viscosity, suggesting that bubble interaction and coalescence following gas injection is to a large extent an inviscid phenomenon for these liquids, many aspects of which can be accounted for without recourse to wake effects or other viscosity-dependent ingredients of some current models.

Keywords: Bubble generation; Coalescence; Potential flow

1. Introduction

The generation of bubbles by injection of a gas into a liquid at rest is an important and much studied problem. Extensive research has been summarized in a variety of models that address the many facets of the problem with different levels of detail; see Refs. [1–5] for reviews. The conceptually simplest models are based on a balance of the forces acting on a bubble of assumed shape (Refs. [6–8], among others). These models clearly show the existence of a regime of low gas flow rate in which the effect of the inertia of the liquid is negligible and the volume of the bubbles is a constant independent of the gas flow rate, and a regime of high gas flow rate in which the effect of the surface tension is negligible and the volume of the bubbles increases as the $6/5$ power of the gas flow rate and is independent of the size of the injection orifice.

The original models of Davidson and Schuler [6] and Ramakrishna et al. [7], which served to establish these results, have been extended to include a variety of effects such as the viscous drag of the bubbles, the flow left by the viscous wake of the preceding bubble, the momentum flux of the injected gas, and the different shapes and apparent masses of the bubble at different stages of its growth. Extensions also include a set of ad hoc criteria to account for the interference, collision and coalescence of bubbles [9], which are observed to occur at high flow rates and eventually

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lead to nonperiodic and chaotic regimes of bubble generation [10]. More sophisticated nonspherical models [11–15] postulate equations of motion for each element of the bubble surface, whose shape changes continuously during the growth and detachment. These models rely to varying degrees on solutions for the potential flow of the liquid [16]. Oguz and Prosperetti [17] numerically computed this flow using a boundary element method and described in full detail the growth and detachment of a single bubble at the end of a tube in different cases of interest, finding good agreement with high speed video visualizations (see also Ref. [18]).

This paper focuses on time periodic bubbling regimes featuring coalescence of two or more bubbles in a strictly inviscid liquid. Though the bubble generation process ceases to be periodic when the flow rate is increased to sufficiently high values, these more complex regimes will not be discussed here. Instead, the purpose of the work is to examine to what extent coalescence at moderate gas flow rates can be described in the framework of potential flow theory. In this respect, the work is an extension of those of Refs. [17] and [18] to include bubble coalescence. The main result is that potential flow computations suffice to describe many aspects of coalescence, without recourse to any wake effect or other effects related to the viscosity of the liquid.

Attention will be restricted to the simplest case of injection of a constant flow rate of a gas through a single circular orifice at the bottom of an inviscid liquid at rest. Fig. 1 is a sketch of the process. The gas will be treated as incompressible, with a density negligibly small compared with the density of the liquid. The only parameters of the problem are then the radius of the orifice, a, the density of the liquid, ρ, the liquid–gas surface tension and the contact angle of the surface with the bottom, σ and θ, the gas flow rate, Q (volume of gas injected per unit time), and the acceleration due to gravity, g. The dimensional parameters can be grouped into a Bond number and a Weber number

$$B=\frac{\rho g a^2}{\sigma} \quad \text{and} \quad We=\frac{\rho Q^2}{\sigma a^3}. \tag{1}$$

2. Formulation

The flow induced in the liquid by the train of bubbles issuing from the orifice of Fig. 1 is irrotational if the viscosity of the liquid is neglected. The velocity potential, φ such that $\boldsymbol{v}=\nabla\varphi$, satisfies the Laplace equation

$$\nabla^2\varphi=0 \tag{2}$$

in the liquid, to be solved with the conditions

$$\frac{Df_i}{Dt}=0, \tag{3}$$

$$\frac{D\varphi}{Dt}=\frac{1}{2}|\nabla\varphi|^2-p_{g_i}-Bx+\nabla\cdot\boldsymbol{n}_i \tag{4}$$

at the surfaces of the bubbles; $\partial\varphi/\partial x=0$ at the horizontal bottom ($x=0$); and $\nabla\varphi\to 0$ at infinity. Here $f_i(\boldsymbol{x},t)=0$ is the equation of the surface of the i-th bubble, with $i=0$ denoting the bubble growing at the orifice and $i=1,2,\ldots$ denoting the bubbles detached previously. These surfaces are to be found as part of the solution. Distances and times are nondimensionalized with the radius of the orifice a and the capillary time $(\rho a^3/\sigma)^{1/2}$. x is the dimensionless height above the bottom, $D/Dt=\partial/\partial t+\boldsymbol{v}\cdot\nabla$ is the material derivative at points of the bubble surfaces, $\boldsymbol{n}_i=\nabla f_i/|\nabla f_i|$, and p_{g_i} is the pressure of the gas in the i-th bubble referred to the pressure of the liquid at the bottom far from the orifice and scaled with σ/a. These pressures are functions of time which are determined by the conditions that the volume of the growing bubble ($i=0$) increases at a constant rate equal to the volume of gas injected per unit time (Q), and the volumes of the detached bubbles ($i=1,2,\ldots$) do not change with time. In dimensionless variables, these conditions read

$$\int_{\Sigma_0}\boldsymbol{v}\cdot\boldsymbol{n}_0\,\mathrm{d}A=We^{1/2} \quad \text{and} \quad \int_{\Sigma_i}\boldsymbol{v}\cdot\boldsymbol{n}_i\,\mathrm{d}A=0, \quad i=1,2,\ldots, \tag{5}$$

where the integrals extend to the surfaces of the bubbles.

An additional condition is needed at the contact line of the growing bubble with the solid. Here the contact line will be taken to coincide with the edge of the orifice when the angle of the liquid–gas surface with the horizontal is larger than the contact angle (i.e. when $-n_{x_0}<\cos\theta$, where n_{x_0} is the vertical component of the unit normal $\boldsymbol{n}_0$ to