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Darcy–Brinkman flow past a two-dimensional screen

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ABSTRACT

The flow past a screen composed of periodic slats in a plane is studied. The method of eigenfunction expansions and point match is used to solve the Darcy–Brinkman equations. The velocities, pressures and resistances are determined for the flow in three orthogonal directions. Aside from screen geometry, the flow is governed by a porous media parameter k which is zero for pure viscous flow. The fundamental case for the flow over a single slat is then extrapolated. It is found that for $k = 0$ the Stokes paradox occurs and the drag rises singularly as k is increased from zero.

1. Introduction

The flow past a screen is important in biological membrane and industrial filtering processes. Due to the small velocity in the crevices, the inertial effects can be ignored and creeping flow assumptions are valid. Analytical solutions are difficult for viscous flow even in the Stokes limit. For thin screens with negligible thickness, Roscoe [1] used an electrostatic potential method to solve for the Stokes flow through an elliptic hole. Roscoe's transform was applied to the slow viscous flow through periodic two-dimensional slits by Hasimoto [2], who was able to express the solution in closed form. Wang [3] used the Roscoe transform semi-analytically for the solution to viscous flow through an array of holes.

In this paper we are concerned with the flow past a thin screen embedded in a porous medium. We shall see later that there are some fundamental differences between Darcy–Brinkman flow and Stokes flow.

The flow in a porous medium is traditionally approximated by the Darcy equation, where the mean velocity is proportional to the pressure gradient, resulting in potential flow. Brinkman added a viscous term so that the no slip condition on solid surfaces can be applied. The Darcy–Brinkman equation is [4–6]

$$\nabla p' = \mu_e \nabla^2 \vec{v} - \frac{\mu}{K}\vec{v} \tag{1}$$

where p' is the pressure, μ_e is the effective viscosity of the matrix, $\vec{v}$ is the velocity vector, μ is the viscosity of the fluid, and K is the permeability. This equation is well accepted for porous media of high porosity such as fiberglass wool. The Darcy–Brinkman equation reduces to Darcy equation when $K \to 0$ and to the Stokes equation when $K \to \infty$. In both limits the problem can be simplified by a velocity potential.

However, for the general Darcy–Brinkman equation, all potential methods fail. We shall use semi-analytic eigenfunction expansions and point match to solve the problem.

Since Eq. (1) is linear, any uniform flow towards a screen can be separated into three independent flows: the flow normal to a screen (Fig. 1(a)), the flow parallel to a screen but still normal to the slats (Fig. 1(b)) and the parallel flow parallel to the slats (Fig. 1(c)). In each case the flow is two dimensional, i.e. depends on x, y directions only. Note that if the Darcy equation is used in the last two cases, the screen would have no effect on the flow.

2. The flow normal to the screen

The top of Fig. 1(a) shows the two-dimensional thin screen. The width of the slats is $2L$ and the period is $2bL$. Cartesian axes are placed at the middle of a slat as shown. We normalize all lengths by L, velocities by the velocity at infinity U, the pressure by $\mu_e U/L$. For the flow normal to the screen, the normalized cross section is shown at the bottom of Fig. 1(a). Define a stream function ψ normalized by UL which satisfies continuity, where the Cartesian velocity components are $(\psi_y, -\psi_x)$. Eq. (1) then becomes

$$p_x = \nabla^2 \psi_y - k^2 \psi_y, \tag{2}$$

$$p_y = -\nabla^2 \psi_x + k^2 \psi_x \tag{3}$$

where

$$k^2 = \frac{\mu L^2}{\mu_e K} \tag{4}$$

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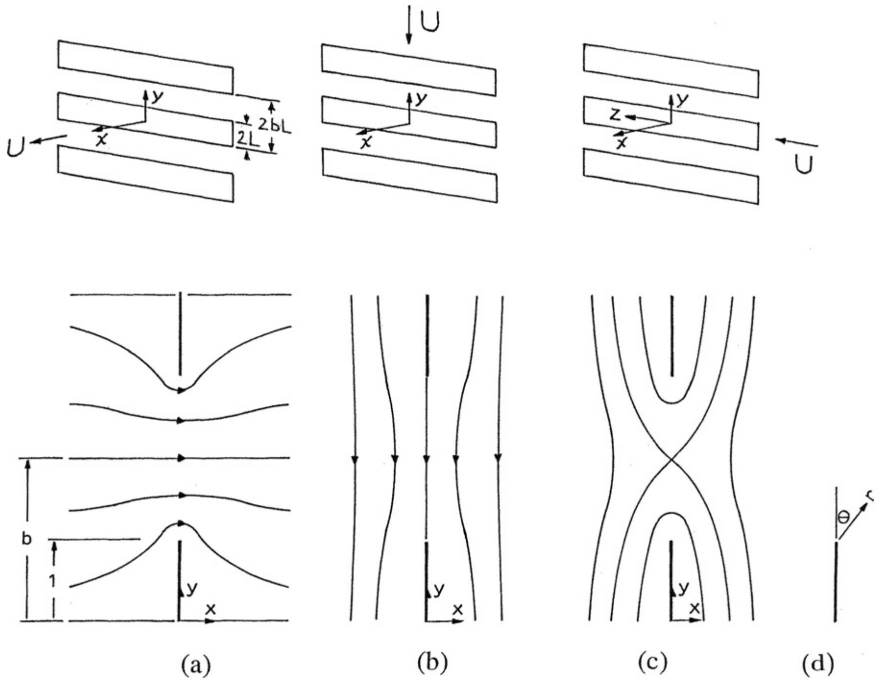

Fig. 1. The top figures show the three orthogonal flow directions across the screen. The bottom figures are the cross sections of each case. (a) The flow is normal to the screen. The mean flow is in the x direction. Some streamlines are shown in the cross section. (b) The flow is parallel to the screen, but normal to the slats. The mean flow is in the $-y$ direction. Streamlines are shown. (c) Parallel flow is parallel to the slats. The flow is in the z direction. Constant velocity lines are shown. (d) Coordinates in the vicinity of an edge.

is an important non-dimensional parameter characterizing the porous medium. Eliminating pressure from Eqs. (2), (3) yields

$$\nabla^2(\nabla^2 - k^2)\psi = 0. \tag{5}$$

Due to symmetry, we need to consider only the strip $x \geqslant 0$, $0 \leqslant y \leqslant b$ which is our computational domain. The symmetry boundary conditions are

$$\psi = 0, \quad \psi_{yy} = 0 \quad \text{on } y = 0, \tag{6}$$

$$\psi = b, \quad \psi_{yy} = 0 \quad \text{on } y = b, \tag{7}$$

$$\psi_x = 0, \quad \psi_{xxx} = 0 \quad \text{on } x = 0,\ 1 < y \leqslant b. \tag{8a,b}$$

The no-slip boundary condition is

$$\psi_x = 0, \quad \psi = 0 \quad \text{on } x = 0,\ 0 \leqslant y < 1. \tag{9a,b}$$

At infinity the flow is uniform, such that $\psi_y = 1$. The solution to Eqs. (5)–(7), (8a), (9a) is

$$\psi = y + \sum_{n=1}^{\infty} A_n \sin(\alpha_n y)\left(e^{-\alpha_n x} - \frac{\alpha_n}{\sqrt{\alpha_n^2 + k^2}} e^{-\sqrt{\alpha_n^2+k^2}x}\right) \tag{10}$$

where $\alpha_n = n\pi/b$ and A_n are coefficients to be determined. The remaining boundary conditions are satisfied by point match. Truncate the series to N terms and consider N equally-spaced points $y_j = b(j - .5)/N$, $j = 1, \dots, N$. Then Eq. (8b) becomes

$$\sum_{n=1}^{N} A_n \sin(\alpha_n y_j)\alpha_n = 0, \quad 1 < y_j \leqslant b. \tag{11}$$

Eq. (9b) yields

$$\sum_{n=1}^{N} A_n \sin(\alpha_n y_j)\left(1 - \frac{\alpha_n}{\sqrt{\alpha_n^2 + k^2}}\right) = -y_j, \quad 0 \leqslant y_j < 1. \tag{12}$$

From Eqs. (11), (12) the coefficients A_n are inverted. Convergence is fairly fast. In general $N = 100$ is adequate for a three figure accuracy in ψ. From Eqs. (2), (3) pressure is integrated to be

$$p = -k^2 x + k^2 \sum_{1}^{N} A_n \cos(\alpha_n y) e^{-\alpha_n x} + c. \tag{13}$$

The integration constant c is determined by setting the average pressure at $x = 0$, $1 < y \leqslant b$ to zero. Thus

$$c = \frac{k^2}{b-1} \sum_{1}^{N} A_n \frac{\sin \alpha_n}{\alpha_n}. \tag{14}$$

Typical streamlines are shown in Fig. 2(a) and the corresponding pressure distribution is shown in Fig. 2(b). Note the convergence of the pressure lines at the edge of the slat. Such singularity in pressure is analyzed in Section 5. The value $\Delta p = |2c|$ also represents the additional pressure loss due to the screen. Fig. 3 shows for given spacing b, Δp rises with the porous parameter k. When $k \to 0$, the pressure drop approaches that predicted by Hasimoto [2]

$$\Delta p = \frac{2\pi/b}{|\ln[\cos((b-1)\pi/2b)]|}. \tag{15}$$

Our computed numerical values from Eq. (14) agrees with those of Eq. (15) as $k \to 0$.

Of interest is the force on a single slat, which can be isolated by increasing the distance b. Both consideration of momentum difference and integration of pressure difference on a single slat shows the (drag) force is $D = 2b\Delta p$. Since it is not possible to use $b = \infty$ in our computation, the single slat or plate problem is approached as follows. Assuming for large b

$$D \sim a_0 + a_1/b^2 + \cdots. \tag{16}$$

For given k we compute D for large b using an N which guarantees at least 10 collocation points on the slat (up to $N = 2000$). Then D is plotted against $1/b^2$ for various b. A typical result is shown