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Hydrodynamic forces acting on a rigid fixed sphere in early transitional regimes

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Abstract

A spectral – spectral-element code is used to investigate the hydrodynamic forces acting on a fixed sphere placed in a uniform flow in the Reynolds number interval [10–320] covering the early stages of transition, i.e. the steady axisymmetric regime with detached flow, the steady non-axisymmetric and the unsteady periodic regimes of the sphere wake. The mentioned changes of regimes, shown by several authors to be related to a regular and a Hopf bifurcations in the wake, result in significant changes of hydrodynamic action of the flow on the sphere. In the present paper, we show that the loss of axisymmetry is accompanied not only by an onset of lift but also of a torque and we give accurate values of drag, lift and torque in the whole interval of investigated Reynolds numbers. Among other results show, moreover, that each bifurcation is accompanied also by a change of the trend of the drag versus Reynolds number dependence, the overall qualitative effect of instabilities being an increase of drag.

Keywords: Sphere; Wake; Forces; Instabilities

1. Introduction

Large amount of recent numerical and theoretical work [1–5] has been focused on the investigation of instabilities responsible for the transition of the sphere wake to turbulence. It has been shown that this transition is accompanied by a symmetry breaking setting in at the primary, regular, i.e. steady to steady, bifurcation whereby axisymmetry gives way to a plane symmetry with an arbitrarily chosen plane direction. A wide consensus allows to place the critical Reynolds number (based on the asymptotic flow velocity U_∞ and the sphere diameter d, $Re = U_\infty d/\nu$, ν being the kinematic viscosity of the fluid) at $Re_1 = 212$. The transition of the steady regime to an unsteady periodic one is triggered by a Hopf bifurcation at a critical Reynolds number found in [5,6] to be $Re_2 = 273$. The obtained results have a fundamental importance for the understanding of the transition from a steady and axisymmetric regime to a non-axisymmetric and unsteady state of the wake. Relatively little interest has been so far given to the way how the instabilities change the hydrodynamic forces acting on the sphere. In spite of the fact that the knowledge of hydrodynamic forces is not necessary for the understanding of wake instabilities as long as the sphere is kept fixed, the issue of hydrodynamic forces acting on a sphere is of greatest historical and practical interest. Moreover, in the transitional

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(dynamically low dimensional) regimes they provide a good characteristic of the wake as a dynamical system. The calculation of the sphere drag was probably the first, and for a long time the only, problem consisting in solving the full problem of flow equations and hydrodynamic forces. Except for infinitely low particle Reynolds numbers (typically up to $Re = 0.5$) where the Stokes approximation applies [7] and for Reynolds numbers on the order of one, for which the theoretical Oseen's solution is satisfactory, the drag versus Reynolds number laws are empirical results of much experimental data used to extrapolate the only available theoretical results to higher Reynolds numbers. The well known laws [8,9] provide satisfactory agreement with experiments and numerical results up to $Re \approx 200$. Beyond 200, there are two basic reasons why these laws fail to be satisfactory. Firstly, though assuming the flow past the sphere axisymmetric, these laws have been found to present an increasing error beyond $Re = 200$ if compared to axisymmetric numerical simulations (6% of error is stated by [10] for the law established by [8]). Secondly, and more importantly, the empirical laws based on the assumption that the hydrodynamic action of the flow on spherical body is isotropic and are described by a smooth function of the Reynolds number fail to account for well known effects due to axisymmetry breaking and transition to unsteadiness in the sphere wake. The best evidence of anisotropy of the hydrodynamic force is probably due to Johnson and Patel [4] who computed the lift force arising above the primary (axisymmetry breaking) instability threshold at $Re = 250$ and $Re = 300$. This force lies in the symmetry plane selected by the bifurcation and its magnitude was found to be a little less than 10% of the drag at $Re = 250$ and roughly 10% of the drag at $Re = 300$. The onset of unsteadiness results, naturally, in an unsteadiness of drag and lift.

The purpose of the present paper is to provide a systematic study of hydrodynamic forces acting on a fixed sphere in a wide interval of Reynolds numbers starting with the flow detachment going up to the upper bound of the unsteady periodic regime. The numerical simulations are carried out using the spectral – spectral-element code described in Ghidersa and Dušek [5]. Additional validation and accuracy estimations are presented throughout the paper, namely in Sections 3, 5 and 7. They are based on the numerical convergence of investigated physical quantities such as the drag, the axisymmetry breaking threshold and the amplitude of the oscillations in the unsteady periodic regime. The drag in the axisymmetric regime computed between $Re = 20$ and $Re = 210$ is presented in Section 4. The results concerning the steady non-axisymmetric regime are given in Section 6. A special attention is paid to the torque. Results relative to the unsteady regime are given in Section 7.

2. Theory and numerical method

In this section, we present a summary of the mathematical formulation and numerical method used in this study. For more details refer to Tomboulides et al. [3] and Ghidersa and Dušek [5].

2.1. Mathematical formulation

We consider the unsteady flow of an incompressible fluid of density ρ and dynamical viscosity μ around a sphere of diameter d. We use the cylindrical coordinate system (z, r, θ), the *z-axis* being parallel to the uniform inflow direction U_∞. The coordinates are non-dimensionalized with respect to the length scale d. The axial, radial and azimuthal velocities in this system, denoted respectively by (u, v, w), are non-dimensionalized with respect to the inflow velocity U_∞. The pressure p is non-dimensionalized with respect to ρU_∞^2; the dimensionless time is $t = d/U_\infty$.

With these variables, the Navier–Stokes equations are expressed as the following system of equations:

$$\frac{\partial \vec{v}}{\partial t} + (\vec{v} \cdot \vec{\nabla})\vec{v} = -\vec{\nabla} p + \nu \nabla^2 \vec{v}, \tag{1}$$

$$\nabla \cdot \vec{v} = 0 \tag{2}$$

where $\nu = 1/Re$ stands for the inverse of the Reynolds number $Re = U_\infty d\rho/\mu$. For non-swirling axisymmetric flows, the flow field solution of (1) and (2) is independent of θ and has no azimuthal velocity component.

The linear stability analysis of an axisymmetric flow field $(\vec{V}, P)$ consists in introducing an infinitesimal perturbation $(\vec{v}', p')$:

$$\vec{v} = \vec{V} + \vec{v}',$$

$$p = P + p'.$$

This perturbation is expressed via complex eigenmodes $\vec{\Phi}, \Pi$: