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Evolution of a model dune in a shear flow

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Abstract

We present a simplified model for the displacement of a model dune in a constant viscous shear flow over a non-erodible soil. A simplified linear law with a threshold effect (in shear stress) and saturation is used to link the flux of sediments to the shear stress. The asymptotic framework of "Double Deck" (large Reynolds number laminar flow theory) is used for the flow. This method allows the computation of boundary layer separation, and the flow may be further simplified with an analytical relation linking the dune shape to the skin friction. For a given shape, the asymptotic solutions give a good agreement with Navier Stokes computations. Examples of displacement of model dunes are presented. We then obtain a selfsimilar coupled problem, predicting that the velocity of the dune is proportional to $m^{-1/4}$. Computations indicate that there is no dune if the mass of the dune is too small, or if the saturation length is too large, or if the threshold is too small.

Keywords: Boundary layer; Erosion sedimentation

1. Introduction

The study of ripples and dunes formation is attractive and receives a growing interest. It is a difficult problem to predict how a sand dune will emerge from the shore and move on the land, because one has to solve numerically the 3D turbulent Navier–Stokes equations for the air, where the viscosity is changed by the transported sand. Then the transport of sand in this flow has to be solved. Finally, the dune moves due to deposition and erosion and eventually avalanches may appear .... All those phenomena are strongly coupled, so that it is now nearly impossible to perform such computations. First for time consuming reasons, and second, for lack of exact physical knowledge of all the phenomena involved. Of course, the same difficulties arise when we deal with submarine dunes where air is replaced with water.

Sauermann et al. [1], Kroy et al. [2], and Andreotti et al. [3] solved a simplified but realistic coupled model for the displacement of aeolian dunes. Here, on one hand, we present some severe simplifications compared to their work, which allow us to obtain a simplified model problem that we go on to solve numerically. We put neither gravity effect nor avalanche effect. But, on the other hand, the key point in our model is that we make no approximation with

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respect to the re-circulation bubble (they oversimplified the separation bulb). This is possible because the asymptotic framework we used allows boundary layer separation. Of course what we then call a "dune" or a model dune, is only the result of our simplified mathematical model and is quite far from reality. To a certain extent, it may be a first step in the direction of the study of small centimetric dunes in water such as Hersen et al. [4].

In Section 2.1, we recall the most simple way to obtain the flux of materials over an erodible soil, hence linking the saturated flux of materials (sand, sediments, etc.) to the shear stress τ. Two ingredients are retained namely, the existence of a threshold under which no sediments are moved, and a saturation length for the flux before it attains a saturated value. In Section 2.2, we assume that the basic flow is laminar and steady; namely, we assume that it is a simple shear flow (the velocity increases linearly with altitude). We assume that the dune is small in height so that its slope is small (compared to one). This approximation allows us to derive asymptotic solutions of the Navier–Stokes equations in the limit of large Reynolds numbers ($Re = \infty$). This is the so-called Triple Deck theory, see Stewartson and Williams [5], Neiland [6] (more precisely the Double Deck theory: Smith [7]). We note that Fowler [8] obtained a very similar description in over-simplifying a turbulent boundary layer. We compute the flow over a given bump with a Navier Stokes solver (CASTEM), and good agreement is found with the Double Deck computations. Then we present the coupled problem in Section 3, and solve it in Section 4. Following some examples of resolutions leading to the displacement at constant velocity of a "dune" in our simplified framework (either with non-linear or linear fluid solution), we propose a similar system of equations depending only on two final parameters. These parameters are the threshold value of the skin friction and a combination of the saturation length and the mass. Again, we find that there is a critical size under which no "dune" exist and that the larger the "dune" is, the slower it moves.

2. Basic equations

2.1. The erodible bed: relations between q and the flow

Du Boys [9] was one of the first to work on and to present a review of the subject. He understood that a critical value of the flow velocity must be reached in order to create a saturated flux of materials q_s. Since then (see Yang [10] who presents the other pioneering works such as: Exner (1925), Shields (1936), and Bagnold (1941), etc., as well as a comprehensive modern review), many other laws have been proposed, for which in general, the saturated flux of materials transported by the flow per unit width is an increasing function of the skin friction (or equivalently the shear stress: τ):

$$q_s = E\tau^a \varpi(\tau - \tau_s)^b, \tag{1}$$

where the threshold function ϖ is such that if $(\tau - \tau_s) > 0$, then $\varpi(\tau - \tau_s) = (\tau - \tau_s)$ else, $\varpi(\tau - \tau_s) = 0$. The coefficients a, b, E and τ_s depend on the modelling. The latter, when adimensionalised, is known as the threshold Shield number $\theta_s = \tau_s/(\rho_p - \rho)gd$; ρ and ρ_p are fluid and particle density, respectively, d is average particle diameter, and g is gravity. Flux arises when shear stress is high enough.

Du Boys [9] pioneering theoretical law corresponds to $a = b = 1$; Charru and Mouilleron-Arnould [11] used a law issued from (laminar) resuspension theory corresponding to $a = 0$, $b = 3$; Charru, Mouilleron-Arnould and Eiff [12] from laminar experiments in a annular channel, obtained $a = 1$, $b = 1$; in a turbulent water flow Sumer and Bakioglu [13] use $a = 1/2$, $b = 1$, Peter–Meyer (see Fredsøe and Deigaard [14]) use $a = 0$, $b = 3/2$; in a laminar water flow Blondeaux [15] uses $a = 0$, $b = 4.28$, in an eolian context Kroy et al. [2] use $a = 3/2$, $b = 0$. Those are the typical values.

The global scaling of the flux is a length time a velocity. In order to analyse experiments, the length used is mainly the grain diameter itself d. The velocity is mainly taken to be the settling velocity ($(\rho_p - \rho)gd^2/\mu$ in laminar flows, $\sqrt{(\rho_p - \rho)gd}$ in turbulent flows). For instance, laminar experiments of [12] give:

$$q_s = 0.85\frac{(\rho_p - \rho)gd^2}{18\mu}\left(\frac{\tau}{(\rho_p - \rho)gd}\right)\varpi\left(\frac{\tau}{(\rho_p - \rho)gd} - 0.12\right), \tag{2}$$

whereas the usual Peter–Meyer law [14] for turbulent flows reads:

$$q_s = 8\sqrt{(\rho_p - \rho)gd^3}\,\varpi\left(\frac{\tau}{(\rho_p - \rho)gd} - 0.047\right)^{3/2}. \tag{3}$$