Experimental Thermal and Fluid Science 64 (2015) 125–133

Contents lists available at ScienceDirect

Experimental Thermal and Fluid Science

journal homepage: www.elsevier.com/locate/etfs

Generalized Reynolds number and viscosity definitions for non-Newtonian fluid flow in ducts of non-uniform cross-section

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ARTICLE INFO

Article history:
Received 10 October 2014
Received in revised form 6 February 2015
Accepted 7 February 2015
Available online 14 February 2015

Keywords:
Non-Newtonian
Viscosity
Generalized Reynolds number
Non-uniform cross-section
Power Law

ABSTRACT

This work presents and experimental study of the generalization method of the Reynolds number and the viscosity of pseudoplastic fluid flow in ducts of non-uniform cross-section. This method will permit to reduce 1 degree of freedom of hydrodynamical and thermal problems in those ducts. A review of the state of the art has been undertaken and the generalization equation proposed for ducts of uniform cross section has been used as a starting point. The results obtained with this equation have not been found satisfactory and a new one has been proposed.

Specifically, the procedure has been developed for two models of scraped surface heat exchanger with reciprocating scrapers. For both models, the scraper consists of a concentric rod inserted in each tube of the heat exchanger, mounting an array of plugs that fit the inner tube wall. The two models studied differ in the design of the plug.

The procedure to perform the generalization method out of experimental data is accurately detailed in the present document.

1. Introduction

Many fluids in the food and chemical or petrochemical industries are non-Newtonian. In such applications the determination of parameters such as the friction factor and the Nusselt number is necessary for the calculation of pressure losses and heat transfer rates or temperature distributions in heat exchangers. This can be achieved experimentally or theoretically by solving the appropriate transport equations for typical common geometries (circular ducts, flat ducts, etc.). An important characteristic of these fluids is that they have large apparent viscosities; therefore, laminar flow conditions occur more often than with Newtonian fluids.

Pseudoplastic fluids are the most common non-Newtonian fluids in the process industry Chhabra and Richardson [5]; Cancela et al. [4]. For this fluids, in a certain range of shear stress, the viscosity decreases as shear stress increases. To describe this behaviour, various mathematical models can be used. Among them, the Power Law model is widely used because of its simplicity. The model can be used to explain the viscosity of a specific fluid in a limited range of shear rates. The Power Law model (Eq. (1)) has two parameters: the flow behaviour index n and the flow consistency index m. Thus, the hydrodynamic and thermal problems have one additional degree of freedom, which increases their complexity.

$$\tau = m\gamma^n \tag{1}$$

For example, let us consider the study of pressure drop in fully developed flow in pipes for forced convection. The list of significant variables can be $p_L = \Psi(D, u_b, \rho, m, n)$. Through the Pi Theorem the problem simplifies to three non-dimensional numbers $f = \Psi(Re, n)$. Consequently, the relation between Re and the friction factor will be different for fluids with different n. With the previous list of variables, the Reynolds number for Power Law fluids would be,

$$Re_b = \frac{\rho u_b^{2-n} D^n}{m} = \frac{\rho u_b D}{\mu_b} \tag{2}$$

where viscosity would be defined by $\mu_b = m(u_b/D)^{n-1}$. Other viscosity definitions, with the same dimensional equations, are possible and will be more useful for the study of pressure drop in heat exchangers.

Metzner and Reed [24] where the first to use the so called generalization method. They analytically obtained the relation between the friction factor f and the Reynolds number Re_b for the fully developed laminar flow in a pipe. Then, they defined a

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http://dx.doi.org/10.1016/j.expthermflusci.2015.02.005
0894-1777/

Nomenclature

m	flow consistency index (rheological property) (Pa s^n)
D	inner diameter of the heat exchanger pipe (m)
D_v	inner diameter of the viscometer pipe (m)
d	diameter of the insert device shaft (m)
D_h	hydraulic diameter $D_h = D - d$ (m)
L_p	pipe length between pressure ports of test section (m)
L_v	viscometer pipe length between pressure ports (m)
N	number of measures for each experiment
P	pitch of the insert devices (m)
p	pressure (Pa)
p_L	pressure drop by length unit (Pa/m)
Q	flow rate (m^3/s)
S	main cross-section (m^2)
u_b	bulk velocity (m/s)

Dimensionless numbers

n	flow behaviour index (rheological property)
Re	Reynolds number, $Re = \rho u_b D_h/\mu$
f	Fanning friction factor, $f = \Delta p D_h/2L\rho u_b^2$
ξ	pressure drop constant dependent on the duct geometry
a to e	correlation constants

Greek symbols

α	exponent of Re_b in experimental correlations (s^{-1})
γ	shear rate (s^{-1})
μ	fluid viscosity (exact definition indicated by the subindex) (Pa s)
ϕ	function of n
Ψ	unknown function (kg/m^3)
ρ	fluid density (kg/m^3)
τ	shear stress (Pa)

Subscripts

b	Reynolds number or viscosity defined by Eq. (2)
g	Reynolds number or viscosity defined by Eqs. (18) and (19)
MR	defined by Metzner and Reed [24] (Eqs. (4) and (5))
DL	defined using the equation from Delplace and Leuliet [6] (Eqs. (6) and (7))
$\xi = an$	generalization based on pressure drop in annulus, where ξ is obtained from Kozicki et al. [20])
$\xi = exp$	ξ in Eqs. (6) and (7) is obtained by experimental correlation
v	belonging to the viscometer
w	at the inside pipe wall

new Reynolds number Re_{MR}, being the one which multiplied by the friction factor gave the same result that the one given by a Newtonian fluid.

$$f \times Re_{MR} = 16 \tag{3}$$

$$Re_{MR} = \frac{\rho u_b^{2-n} D^n}{m 8^{n-1}((3n+1)/(4n))^n} = \frac{\rho u_b D}{\mu_{MR}} \tag{4}$$

being the generalized viscosity for the flow in pipes

$$\mu_{MR} = m\left(\frac{u_b}{D_h}\right)^{n-1} 8^{n-1}\left(\frac{3n+1}{4n}\right)^n \tag{5}$$

Kozicki et al. [20] obtained a relation between friction factor and Reynolds number for various simple geometries (circular pipes, parallel plates, concentric annuli and rectangular, isosceles triangular and elliptical ducts) as a function of two parameters. Afterwards, Delplace and Leuliet [6] reduced those parameters to one. Therefore, the definition of Metzner and Reed [24] can be applied to geometries with uniform cross-section as a function of a single geometric constant.

$$Re_{DL} = \frac{\rho u_b^{2-n} D_h^n}{m \times \xi^{n-1}\left(\frac{24n+\xi}{(24+\xi)n}\right)^n} \tag{6}$$

$$\mu_{DL} = m\left(\frac{u_b}{D_h}\right)^{n-1} \xi^{n-1}\left(\frac{24n+\xi}{(24+\xi)n}\right)^n \tag{7}$$

$$f \times Re_{DL} = 2\xi \tag{8}$$

For duct geometries of uniform cross-section different from the ones studied by Kozicki et al. [20], similar relations can be obtained either experimentally or numerically. This simplification leads to significant reduction in the study cases of a particular problem. This has been called a generalization method because it allows to express the pressure drop behaviour of Newtonian and non-Newtonian fluids with a single curve. Consequently, the Reynolds number and viscosity defined by this method are known as the *generalized Reynolds number* and the *generalized viscosity* [19,5]. Besides, the generalized viscosity can be used to generalize other dimensionless numbers such as the Prandtl number in non-isothermal flows [15,6].

The described method has been used by many authors until recent days [13,14,12]. But, as mentioned before, it can only be applied to ducts with uniform cross-section, where the shear-stress at the wall is uniform along the duct.

Enhanced heat exchangers EHE [16,22] are widely used in the process industry in order to enhance heat transfer and they work often with non-Newtonian fluids. Webb [30] classified enhancement techniques into active, if they require external power, and passive, if they do not. Active techniques as scraped surface heat exchangers SSHE are specially designed to avoid fouling and enhance heat transfer. This last kind of enhanced heat exchanger is specially useful for the work with non-Newtonian fluids because of their high viscosity [25]. In most EHE designs, specially in SSHE, the cross-section varies along their length or else the cross-section is uniform but complex and has not previously been studied. Therefore, the generalization method must be based on experimental or numerical results and it is not straightforward.

To overcome this inconvenience, most authors have considered their geometry to be very similar to one of the simple uniform cross-section geometries studied by Kozicki et al. [20] or Metzner and Reed [24]. This is the case of corrugated pipes or pipes with wire coil or twisted tape inserts. Manglik et al. [21]; Oliver and Shoji [26]; Patil [27]; Martínez et al. [23] took this option for their studies of passive EHE performance with non-Newtonian fluids and Igumentsev and Nazmeev [17] did so for his study of SSHE. However, there are complex geometries where this assumption is not valid at all. For those cases, Delplace and Leuliet [6] proposed the use of experimental methods to obtain the value of ξ. Based on the previous research of Rene et al. [28], they proposed to use $\xi = 56.6$ for a plate heat exchanger type. Afterwards some other