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Meshless inverse method to determine temperature and heat flux at boundaries for 2D steady-state heat conduction problems

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ABSTRACT

Inverse determination of temperature and heat flux at an inaccessible surface of a solid has been widely employed in recent years. In this paper, a meshless inverse method, i.e. the method of fundamental solution (MFS), has been developed to determine the temperature field and hence the local boundary temperature and heat flux distributions for a 2D steady-state heat conduction problem based on temperature measurements at interior sample points in the wall of the boundary. A case study showed that MFS predicts the boundary temperature and heat flux with about the same accuracy as the Beck's function specified method but consumes significantly less computing time. Error analysis was carried out regarding uncertainty in location and accuracy of temperature measurement to demonstrate the reliability of the proposed method.

1. Introduction

The inverse heat conduction problem (IHCP) is used to determine temperature and heat flux at boundaries utilizing temperatures measured at interior points. Various methods of solving the IHCP have been reviewed in [1]. The mesh-based methods include, for example, finite difference method (FDM) [2,3], finite element method (FEM) [4,5], finite volume method (FVM) [6,7] and boundary element method (BEM) [8,9]. FDM, FEM and FVM are used to discretize the whole domain, whilst BEM to boundary only. Recently, the meshless methods have been developed as alternatives to classical discretization methods including the Kansa's method based on radial basis functions [10], method of fundamental solution (MFS) [11], local Petrov-Galerkin method [12], boundary knot method (BKM) [13] and Monte Carlo method [14]. These meshless methods require neither domain nor boundary discretization, therefore the computational efficiency can be improved significantly. Optimum procedures have been introduced using the least squares method [15], least squares method with regularization terms [16], conjugate gradient methods [17], steepest descent method [18] and adaptive iterative filter method [19].

The meshless inverse method used in the present work is MFS, which was firstly proposed by Kupradze and Aleksidze in 1960s [20]. Compared with other meshless inverse methods, the implementation of MFS is relatively simple due to the fact that the solution is approximated by linear combination of fundamental solutions corresponding to fictitious heat source points outside the domain considered. This feature makes MFS suitable for solving IHCP even with complicated geometries and guarantees accurate results at small computational cost and consequently has been developed rapidly within the last decade. Hon and Wei [21] successfully developed MFS to solve transient IHCP with implementation of Tikhonov regularization technique and the *L*-curve method to obtain a stable numerical approximation to the solution. They later extended their work to multidimensional IHCP and the effectiveness of this method was verified by several numerical examples [22]. Their work was also extended by Dong et al. [23] for 2D IHCP in an anisotropic medium and the implementation of the truncated singular value decomposition and the *L*-curve criterion to solve the resulting matrix equation. Jin et al. [24,25] proposed a scheme for solving IHCP based on MFS, in conjunction with a regularization method. The efficiency, accuracy, convergence and stability of the scheme were demonstrated by numerical results. They pointed out potential extensions of the method, such as ill-posed Cauthy problems, inhomogeneous problems, inverse source problems associated with other elliptic partial differential equations. Other recent works have further extended the applications of MFS [26–31].

The present work stems from the need for a method to accurately measure local temperature field and heat flux along the flow

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Nomenclature

$\vec{n}$	outwards unit normal vector to the boundary
n_x, n_y	the components of $\vec{n}$ in the x, y direction respectively
n_{sp}	number of heat source
n_{fp}^{Q}	number of field point with known heat flux
n_{fp}^{T}	number of field point with known temperature
Q	superpositioned heat flux
q^*	heat flux excited by heat source x'
r	modular of the distance vector $\vec{r}$
$\vec{r}$	distance vector between field point x and heat source x'
r_x, r_y	components of $\vec{r}$ in the x, y direction respectively
T	superpositioned temperature
T_b	fluid temperature flowing over the bottom surface
T_t	fluid temperature flowing over the top surface
u^*	field point temperature excited by heat source x'
x	field point
x'	heat source

Greek symbols

β	superposition coefficient of the heat source
α_b	heat-transfer coefficient at the bottom surface
α_t	heat-transfer coefficient at the top surface

and channel surface during condensation in microchannels (Wang and Rose [32]). It is very difficult to obtain the local surface temperature and heat flux at such an inaccessible boundary surface. Su et al. [33] shows that available experimental heat transfer data for condensation in microchannels were largely scattered. Vapor-side, heat-transfer coefficients had generally been inferred from overall measurements by subtraction of thermal resistances and/or using "Wilson plot" techniques and therefore have high uncertainty. Rose [34] discussed in detail on heat-transfer coefficients, Wilson plots and accuracy of thermal measurements. However, it is relatively easy to measure the temperatures at interior points in the wall of the boundary and then to determine local surface temperatures and heat fluxes using the inverse method. In order to maintain the generalization in the present study, MFS is proposed to solve a general IHCP using measured interior wall sample temperatures as input. It should be noted that the present method can be applied to a range of heat transfer problems and that the inverse method is not limited to any specific IHCP.

2. Method of fundamental solution (MFS)

A 2D steady-state heat conduction case was analyzed to illustrate MFS as shown schematically in Fig. 1. The actual problem with domain Ω is transformed into an indirect problem in an infinite body and the outer boundary Γ conditions including temperature and heat flux are modeled by assuming fictitious point heat sources collocated on the fictitious boundary Γ'. This general problem in a finite domain can be transformed into one in an infinite plane in order to obtain the solution in an indirect way. The points, where either temperature or heat flux is known, are defined as field points in the domain. The total number of the heat sources is n_{sp} and they are arranged out of the domain in the actual problem. Each of heat sources contributes to the temperature and heat flux at all field points in the domain and at the boundary surfaces. The distance d between the heat source and the point in the domain is also shown in Fig. 1. For a 2D steady-state heat conduction problem, the governing equation with a heat source is given by:

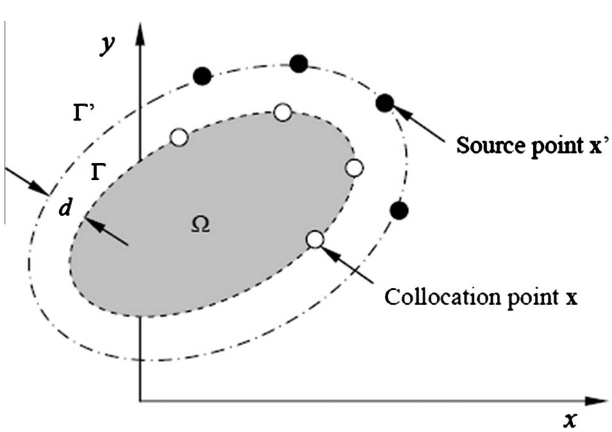

Fig. 1. The method of fundamental solution and fictitious boundary.

$$\nabla^2 u^* = -\Delta(x', x) \tag{1}$$

where u^* represents temperature, x' is the location of the heat source point and x is the field or collocation point. $\Delta(x', x)$ is the Dirac delta function, which is a mathematical description of the point source excitation and satisfies $\int \Delta(x', x)dV = 1$. The solution for Eq. (1) is [35]:

$$u^* = \frac{1}{2\pi}\ln(1/r) \tag{2}$$

where r is the modular of the distance vector $\vec{r}$ between field point x and heat source x'. Heat flux is obtained by:

$$q^* = \frac{\partial u^*}{\partial \vec{n}} \tag{3}$$

where $\vec{n}$ is the outwards unit normal vector to the boundary. Substituting Eq. (2) into Eq. (3) gives:

$$q^* = \frac{-1}{2\pi r^2}(r_x n_x + r_y n_y) \tag{4}$$

where r_x and r_y are the components of $\vec{r}$ in the x and y directions, respectively; n_x and n_y are the components of normal in the x and y directions, respectively.

By the principle of superposition for linear problems, the temperature at field or collocation point can be represented by:

$$T = \sum_{i=1}^{n_{sp}} \beta_i u_i^* \tag{5}$$

where β_i are unknown coefficients of the heat sources (density of the heat source). For field point j, temperature can be written as

$$\sum_{i=1}^{n_{sp}} \beta_i u^*(x_i, x_j) = T_j, (j = 1, \ldots, n_{fp}^{T}) \tag{6}$$

where T_j is the temperature measured, n_{fp}^{T} is the number of points with temperature measured, and

$$\sum_{i=1}^{n_{sp}} \beta_i q^*(x_i, x_j) = Q_j, (j = 1, \ldots, n_{fp}^{Q}) \tag{7}$$

where Q_j is the heat flux and n_{fp}^{Q} is the number of points with heat flux measured. Eqs. (6) and (7) constitute an ill-conditioned linear system of algebraic equations and the unknown coefficients can