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Components of wall shear rate in wavy Taylor–Couette flow

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ABSTRACT

The time-resolved axial and azimuthal components of the wall shear rate were measured as function of Reynolds number by a three-segment electrodiffusion probe flush mounted in the inner wall of the outer fixed cylinder. The geometry was characterized by a radius ratio of 0.8 and an aspect ratio of 44. The axial distribution of the wall shear rate components was obtained by sweeping the vortices along the probe using a slow axial flow. The wavelength and phase celerity of azimuthal waves, axial wavelength of vortices and their drifting velocity were calculated from the limiting diffusion currents measured by three simple electrodiffusion probes.

1. Introduction

The laminar flow in the gap between an inner rotating and outer fixed coaxial cylinders becomes unstable at critical Reynolds number due to centrifugal forces. Toroidal counter-rotating Taylor vortices with an axial wavelength equal approximately to the gap width replace original Couette flow [1]. The critical Reynolds number of the onset of Taylor vortices depends on the radius ratio of the cylinders [2,3]. This number does not depend on the aspect ratio Γ (wavelength versus width of the gap) with the exception of low values of Γ.

At higher rotation rates, the axisymmetric Taylor vortices become instable and azimuthal waves are superposed on them. The onset of azimuthal waves is strongly affected by end effects [4]. Even for an aspect ratio as great as 40, the onset of waves occurred at a Reynolds number greater than that at larger aspect ratios. Jones [5,6] calculated numerically the stability of Taylor vortices for radius ratios in the range $0.5 < \eta < 1$ using the approximation of infinite length cylinder.

Further increase in rotation rate induces azimuthal waves with modulated amplitude, then turbulent flow occurs in cells occupied earlier by vortices and, finally, the turbulent flow spreads throughout the whole gap.

Although the stability of supercritical circular Couette flow has been studied extensively, results for the velocity field of the flow are limited. Using finite-difference method, Fasel and Booz [7] calculated velocity fields of axisymmetric Taylor vortex flow in wide gap, $\eta = 0.5$, for the Reynolds number as high as $100Re_c$. Sengupta et al. [8] used Fluent for calculation of the velocity field of Taylor vortices in an narrower gap, $\eta = 0.8$, for a Reynolds number of 253. Marcus [9] simulated numerically non-axisymmetric wavy vortex flow in $\eta = 0.875$ and up to $15Re_c$.

Akonur and Lueptow [10] used particle image velocimetry for measurements of the azimuthal and radial velocities in latitudinal planes perpendicular to the axis of rotation of wavy Taylor–Couette flow characterized by a radius ratio of 0.81. These measurements were matched to measurements of the axial and radial velocity in several meridional planes with the aim to get time-resolved, three-dimensional, three-component velocity field for wavy Taylor–Couette flow. The results were published for three Reynolds numbers (124.3, 240.8 and 585.5). The PIV technique was also applied by Abcha et al. [11] for the measurements of the axial and radial velocity components of different instability modes in a geometry characterized by the radius ratio 0.8 and aspect ratio 45.9.

A vast number of applications of the Taylor–Couette flow (TCF) as reactor have been proposed, covering the field of catalytic [12], biocatalytic [13], electrochemical [14–16], photochemical [17] and polymerization reactions [18,19] as well as the mass transfer operations, such as counter current extraction [20], tangential filtration [21] and crystallization [22].

Many studies concerning flow patterns in TCF have been accomplished, but the knowledge of local wall shear rates in wavy and higher modes of TCF is only qualitative. This quantity is primordial for the applications like membrane filtration [23], the reactors with catalyst or immobilised enzyme [24] and the bioreactors

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Nomenclature

A_z	amplitude of the axial component of wall shear rate averaged over one wave
a	maximum amplitude of azimuthal waves
d	width of gap between cylinders
d_e	probes diameter
D	coefficient of diffusion
f	frequency
h	height of vortex pair
I_1, I_2, I_3	limiting diffusion currents of three-segment probe
I_4, I_5, I_6	limiting diffusion currents of simple probes
I_{tot}	sum of currents I_1, I_2, I_3
K, b	coefficients in relation $I_{tot} = K\gamma^b$
L	length of gap between cylinders
l_{eq}	equivalent length of probe, $l_{eq} = 0.82d_e$
n	number of waves along the perimeter
Pe	Peclet number, $Pe = \gamma l_{eq}^2/D$
R_1, R_2	radius of the inner and outer cylinders
Re	Reynolds number, $Re = \Omega d R_1/\nu$
Re_{ax}	axial Reynolds number, $Re_{ax} = v_m d/\nu$
R_{44}	autocorrelation function of current 4
R_{45}	correlation of current 5 with respect to current 4
t_{d55}	time of the passage of vortex pair along probe 5
t_{d56}	time of the vortex passage from probes 5 to probe 6
t_{w44}	period of azimuthal waves
t_{w45}	time of wave passage between probes 4 and 5
t	time
ν	kinematic viscosity
v_d	drifting velocity of vortices
v_m	mean velocity of superposed axial flow
z	axial distance
x	critical region of an impinging jet

Greek letters

Γ	aspect ratio, $\Gamma = L/d$
γ	wall shear rate
η	radius ratios, $\eta = R_1/R_2$
Ω_w	angular velocity of azimuthal waves
Ω	angular velocity of the inner cylinder
τ	shear stress
δ	thickness of the concentration boundary layer

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c	transition from Couette flow to Taylor vortex flow
f	averaged over one azimuthal wave (perimeter)
θ	azimuthal
max	maximum
m	mean value over vortex height
min	minimum
r	radial
z	axial

containing shear sensitive cells [25]. The mean values of wall shear rate are known from the torque measurements [26,27] and linear theories [28]. However the wall shear rates of the wavy TCF are functions of space and time. Their fluctuations and especially their maxima are the most important quantity in the above mentioned applications. The experimental methods like Laser-Doppler anemometry [29,30] and PIV [10] do not allow measurements in the wall vicinity which are necessary for correct evaluation of wall shear rates.

The electrodiffusion diagnostics (ED) is a convenient nonintrusive method for measurements of wall shear rates [31]. Using three-segment micro-probes, the components of wall shear rate can be measured [32]. In this work, the axial and azimuthal components of the instantaneous shear rate on the wall of the outer fixed cylinder were measured as a function of rotation rate of the inner cylinder in the geometry characterized by radius ratio $\eta = 0.8$ and aspect ratio Γ about 43. The axial distribution of the wall shear rate components was obtained by sweeping wavy vortices along the fixed probes using a slow axial flow. The method of the vortex displacement by axial flow was already used by Townsend [33] for determining the velocity field of the toroidal vortices by the fixed hot wire anemometers. We also determined the number of azimuthal waves, their phase celerity, axial wavelength of vortices and their drifting velocity from the correlations of the limiting diffusion currents measured by an array of simple probes. This technique enabled us to obtain the mean azimuthal wall shear rate of a vortex pair with an arbitrary wavelength in contrast to the torque measurements which results in the wall shear rate averaged over the whole gap height. Moreover the torque measurements are distorted by the end effects, especially for low depth of liquid.

The electrodiffusion measurements of wall shear rate components for supercritical Couette flow carried out in this paper had several objectives. The first objective was to get detailed space–time cartography of wall shear rate components at several Reynolds numbers. This knowledge can be used for calculation of the instantaneous local wall shear stress via viscosity function. The second objective was to compare the wall shear rate components with the data on velocity fields published by Akonur and Lueptow [10]. The third objective was to verify the theory of inviscid cores surrounded by boundary layers proposed by Batchelor in the paper by Batchelor [26]. The last objective was to verify the numerical data of Jones [5,6] on the critical Reynolds number of the transition to wavy flow.

2. Experimental

The apparatus (see Fig. 1) consisted of an outer cylinder 3 made of a Plexiglas tube with an inner diameter of $R_2 = 62 \pm 0.1$ mm and an interchangeable inner Plexiglas cylinder 4. The inner cylinders had a length of 275 mm and diameter R_1 of 59 and 49.6 mm, respectively. The corresponding radius ratios, $\eta = R_1/R_2$, were 0.95 and 0.8. The larger cylinder was used for the calibration of electrodiffusion probe *in situ*. The inner cylinder was mounted on a stainless steal shaft 2 which had an upper ball bearing and bottom polyamide sliding bearing. The shaft was driven by a stepping motor with a step of 0.9° and a gear box with slow-down ratio 1:9. There was a plastic clutch between the shaft and gear box which also served as electrical insulation. The revolutions were controlled by a computer directly from the measuring software.

After filling the gap between the cylinders without rotation, the pump was stopped and the rotation rate corresponding to $Re = 80$ was adjusted. When laminar Couette flow was fully developed, Taylor vortices were established at Re equal to 100. Wavy vortex flow was then adjusted either by a sudden step or by increasing slowly the rotational rate (4 rpm/min) until determined Re.

The vortices were then swept by a slow axial flow. The liquid was alternatively pushed and pulled by a syringe with a volume of 100 mL at a rate of 0.0858 mL s^{-1} to an inlet tube in the bottom of the apparatus where it was distributed through four holes beneath the inner cylinder. The mean velocity of the axial flow in