Experimental Thermal and Fluid Science 32 (2008) 1538–1547

Contents lists available at ScienceDirect

Experimental Thermal and Fluid Science

journal homepage: www.elsevier.com/locate/etfs

Comparison of quadratic and power law for nonlinear flow through porous media

Nian-Sheng Cheng *, Zhiyong Hao, Soon Keat Tan

School of Civil and Environmental Engineering, Nanyang Technological University, Singapore 639798, Singapore

ARTICLE INFO

Article history:
Received 28 January 2007
Received in revised form 11 January 2008
Accepted 23 April 2008

Keywords:
Porous media
Darcy law
Inertial effect
Turbulence
Nonlinear flow
Seepage
Power law
Groundwater

ABSTRACT

The quadratic and power laws are two typical formulations that can be used to extend the Darcy law to non-Darcy flows through porous media. Both laws are reformulated in the dimensionless form in this study. They are then evaluated by fitting to experimental data with specified variations in seepage velocity, which were specifically collected for a simplified ordered porous model. The results show that the quadratic law is applicable to both linear and nonlinear flow regimes but the two coefficients vary at different regimes. In comparison, the power law appears not workable if the seepage velocity varies over a wide range. This study also demonstrates that the two parameters included in the power law are generally interrelated, and the relationship derived based on the quadratic law compares well with the experimental results. To help understand the nonlinear behavior associated with the simplified porous model, CFD simulations were also performed to visualize and elucidate localized 3D flow phenomena.

1. Introduction

Non-Darcy flows take place at relatively high seepage velocity through porous media such as rockfills, gravel beds and waste dumps. Modelling of such flows is often required in many disciplines including groundwater, geotechnical and environmental engineering. Empirical and theoretical efforts have been made in past decades to extend the Darcy law to the flow through porous media with significant inertial effects. For engineering applications, several formulations of flow resistance have been proposed [10], of which the most typical examples are the quadratic and power models given in the following form:

$$i_s = au_s + bu_s^2 \quad (1)$$

$$i_s = cu_s^m \quad (2)$$

where i_s = hydraulic gradient; u_s = bulk or superficial seepage velocity; a, b and c = coefficients; and m = power varying from 1 to 2. The four parameters (a, b, c and m) can be evaluated from experimental data.

Eq. (1) is also called Forchheimer equation [2]. It was proposed on the grounds that the seepage-related energy dissipation is taken as a sum of energy losses associated with viscous drag and inertial effects. For the case of creeping flow, the viscous drag is linear and thus the Darcy law prevails, as is represented by the first term on the right-hand-side of Eq. (1). With increasing seepage velocity, deviation from the Darcy law takes place because of flow separation at the lee of particles and/or the onset of turbulence. The deviation, in spite of its origins, finally results in a nonlinear drag, which can be well approximated as a function that is proportional to the squared seepage velocity as given by the second term on the right-hand-side of Eq. (1). Some theoretical attempts have been devoted to associate the nonlinear drag with inertial and/or turbulent effects of viscous flow; relevant pioneering references includes Mei and Auriault [11], Skjetne and Auriault [13], Wodie and Levy [16]. However, the theoretical bases of these attempts are still controversial in the literature (e.g. [2]). Direct numerical simulation appears recently as a practical alternative to visualise the transition of flow from linear to nonlinear behaviour. For example, by solving the Navier–Stokes equations for a two-dimensional disordered porous structure with high porosity, Andrade [1] demonstrated that the incipient departure from the Darcy law that is in agreement with Eq. (1) can be observed in the laminar regime of fluid flow without including turbulence effects.

In comparison with Eq. (1), the power form (Eq. (2)) also works well, sometimes even better, in representing variations in the pressure drop measured for different seepage velocities [3,14]. Cheng and Chiew [6] reported that from data-fitting performed for the entire range of the seepage velocity, which was observed through porous column comprising sand or gravel, the estimated power (m) increased with the dimensionless pore length scale.

Although the quadratic law could be somehow derived from the Navier–Stokes equations with certain arguments [4], both types of

* Corresponding author. Tel.: +65 6790 6936; fax: +65 6791 0676.
E-mail address: cnscheng@ntu.edu.sg (N.-S. Cheng).

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doi:10.1016/j.expthermflusci.2008.04.007

functions are often employed to empirically present data-based correlations. This is because it is still generally unclear that to what content these functions are able to characterize reasonably the physics inherent in the flow phenomena. Other questions worthy of further theoretical efforts include under what conditions the nonlinear term appears, and the viscous drag becomes proportional to the squared seepage velocity. Qualitatively, the above questions about seepage flow may be understood by making an analogy with flow around a spherical object. For the latter case, flow separation behind a particle can occur even prior to the onset of turbulence [15], which explains the deviation of the settling drag from the linear Stokes regime.

On the other hand, for engineering applications, it would be very interesting to understand how the two laws differ from each other because one form of function may be preferred to the other. Some analyses in this respect have been presented, for example, by George and Hansen [8], Niven [12]. George and Hansen [8] theoretically showed that the data-based conversions between the two laws are always possible, but the coefficients estimated from one pair to another (i.e. from a and b to c and m, or from c and m to a and b) is subject to the maximum seepage velocity under consideration. Another conversion study is due to Niven [12], who first numerically generated a database of the hydraulic gradient with the quadratic function and then fitted to the power function. The results so-obtained show that the power law was able to predict the hydraulic gradient within an error of 22%.

In this study, seepage was observed through a laboratory experiment designed with a simplified ordered porous media model. The functional relation of flow resistance is presented on the grounds of dimensional analysis in terms of the friction factor and Reynolds number. The measured results are then fitted to the quadratic and power functions and the variations associated with the parameters included in the functions are examined in detail. In addition, numerical simulations were also conducted to help appreciate relevant micro-flow phenomena localised in the pore domain.

2. Laboratory experiment with ordered porous model

The porous media employed in the experiment were modelled by placing glass beads into a square steel pipe of 3 m in length. The diameter of the glass bead, D, was 16 mm and the cross-section of the pipe was measured as $l^2 = 16.4 \times 16.4\ \text{mm}^2$. The beads were packed closely along the pipe and the bead-to-bead gap was not allowed. However, the movement of beads in the transverse direction was possible because of the small difference (0.4 mm gap) between the bead size and the cross-section dimension, but its effect is considered insignificant. The resulted global porosity was 0.50 and the cross-sectional porosity varied from 1 to 0.25 because of a gradual increase in the cross-sectional area of the flow passage (A_p) from the largest blockage ($x/D = 0$) to the zero blockage ($x/D = 0.5$), where x is a longitudinal distance measured from the centre of a glass bead. The variation of A_p with x for $0 \leqslant x \leqslant D/2$ is given by

$$\frac{A_p}{l^2} = 1 - \pi\left(\frac{D^2}{4l^2} - \frac{x^2}{l^2}\right) \tag{3}$$

This simplified model is free of irregularities induced by random granular configuration and also facilitates control of flows. In addition, it would be possible to explore details of fluid flowing in the pores with the aid of computational hydrodynamic technique. As shown later in this paper, the results obtained differ quantitatively from those available in the literature for conventional porous media with random configuration. However, they provide an alternative insight into the transition between the linear and nonlinear flow domains, and also a well-defined physical model for assessing how well the transition can be described with the quadratic and power functions.

To have a wide range of the Reynolds number, both clear water and water–glycerine mixture were used as the fluid media. The viscosity of the mixture which varied with temperature was measured using a piston-type viscometer manufactured by Cambridge Applied Systems. A few probes applied for different ranges of viscosities were employed. The temperature-dependence of viscosity was first calibrated for a range of temperatures and also fitted empirically using exponential functions. The so-obtained calibration curves were then applied to test cases with individual temperatures taken. The pipe flow was driven using a submergible pump. The error in measuring the pressure drop was less than 2% either using a water-based manometer or mercury-based manometer.

Altogether four series of experiments comprising 242 runs were completed. The test conditions are summarized in Table 1. Four kinds of fluids (G00, G50, G70, and G80) were prepared by setting the concentration of glycerine roughly at 0%, 50%, 67% and 80%, respectively. The use of the low-molecular weight fluids, glycerine and its aqueous solutions, implies that the experiments done were under the condition of Newtonian fluids. The variations of the hydraulic gradient with the seepage velocity are plotted in Fig. 1. The average flow velocity varied from 0.004 to 0.282 m/s. The use of glycerine–water mixture made available a range of fluid kinematic viscosity (ν) from 0.7 to 35 cSt ($10^{-6}\ \text{m}^2/\text{s}$). As a result, the Reynolds number ($=u_s D/\nu$) varied from 2 to 5550, where u_s is the superficial seepage velocity and D is the bead diameter. The observed flows covered the Darcy-type linear regime and also inertia-dominant nonlinear regime.

3. Dimensional analysis

Resistance of flows through porous media can be investigated by making an analogy with that considered in conventional pipe flows because both flows are boundary-confined. Several attempts in this respect have been reported in the literature [2], which involve various geometric simplifications of porous media. These are not replicated in this study. Instead, the consideration presented here only engages two parameters, friction factor and Reynolds number. By drawing an analogy with pipe flows, these two parameters can be redefined by including effects of porous media, so that the flow resistance relationship can be presented in a way similar to the Moody diagram, which is often included in fluid mechanics books.

Here, we use f_p and Re_p to denote the friction factor and Reynolds number for pipe flows, respectively. The relation of f_p–Re_p varies generally with the relative roughness length, but becomes unique for hydrodynamically smooth pipes. In this study, it is assumed that the boundary of interstitial flows is hydrodynamically smooth so that the roughness effect is ignored. This is acceptable by noting the experimental condition described in the previous section. By definition, f_p is proportional to $(u^*_p/U_p)^2$ and $Re_p = U_p R_p/\nu$, where $u^*_p = (gR_p i_p)^{0.5}$, R_p = hydraulic radius, U_p is the average velocity through the pipe, ν = kinematic viscosity of fluid, i_p = hydraulic gradient and g = gravitational acceleration.

In the following, both f_p and Re_p are modified to apply for the case of the interstitial flow in porous media. Being differentiated from the parameters used for pipe flows, those related to seepage flow are denoted with the subscript, s. First, the average velocity U_s is computed with respect to the pore space. If the average porosity is ε, U_s is given by

$$U_s = \frac{u_s}{\varepsilon} \tag{4}$$