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Influence of thermal radiation on contaminated air and water flow past a vertical wavy frustum of a cone☆

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ABSTRACT

This paper aims to present detailed numerical solutions of natural convection flow of contaminated air and water along the vertical wavy frustum of a cone. This investigation is particularly conducted to characterize the flow behavior of spherical particles suspended in radiating Newtonian liquids. The governing equations are converted into dimensionless equations by using a set of suitable continuous transformations and solved through the implicit finite difference method. Comprehensive flow formations of the gas and particle phases are given with the aim to predict the enhancement of heat transport across the heated wavy frustum cone. It is recorded that the air–metal mixture can promote the rate of heat transfer drastically whereas water–metal mixture contributes a little in this regard.

1. Introduction

The analysis of the flow of fluids with suspended particles or gas–particle mixture has received notable attention due to its practical applications in various problem of atmospheric, engineering, and physiological fields. Typical examples occurring in nature are dust storms, forest-fire smoke, and the dispersion of the solid pollutants in atmosphere. In addition, solid rocket exhaust nozzles, fluidization in chemical reactors with gas–solid feeds, blast waves moving over the Earth's surface, fluidized beds, environmental pollutants, petroleum industry, purification of crude oil, physiological flows, and other technological fields (see [1]) are some of the practical problems where the dusty viscous flow found its applications. Other important applications involving dust particles in boundary layers include soil salvation by natural winds, lunar surface erosion by the exhaust of a landing vehicle, and dust entrainment in a cloud formed during a nuclear explosion. In this regard, Farbar and Morley [2] initially analyzed the gas–solid mixtures experimentally in circular tube and reported that there is a considerable enhancement in the rate of heat transfer by adding solids to the gas flowing at constant rate. Saffman [3] later formulated the governing equations for the flow of dusty fluid and obtained some results regarding stability of the laminar flow. Since then, many researchers, for example [4–8], attempted to solve such two-phase models numerically or analytically and highlighted the characteristics of gas–particle flow systems in various important physical situations.

It is found that the problem of natural convection flow over a frustum of a cone without transverse curvature effect (i.e., large cone angles when the boundary layer thickness is small compared with the local radius of the cone) has not been treated in the literature, even though the problem for a full cone has been treated quite extensively [9–15]. However, a very limited work has been found in the literature for the truncated cone [16–19]. It is, therefore, the purpose of this paper to present an analysis of the effects of radiative heat transfer on natural convection flow of a two-phase dusty fluid flowing along a vertical wavy frustum of a cone. It is noteworthy that cone shape geometries are commonly used in multiple industry applications to capture and remove debris prior to putting a pipeline into production service. Due to the consequences of dusty flows in various technological fields, it is aimed in the present paper to investigate the effect of dusty fluid along the truncated cone. For this, the governing boundary layer equations for the two-phase/dusty fluid are reduced to a convenient form by the introduction of the primitive variable formulations. Some practical scenario's like

- metal particles in gas: $D_\rho = 10^4$, $\Pr = 0.7$, $\gamma = 0.45$
- metal particles in water: $D_\rho = 10$, $\Pr = 7.0$, $\gamma = 0.1$

are chosen to present the solutions in the form of wall shear stress, heat transfer rate, streamlines, and isotherms by varying several controlling parameters. The above values are taken from the study of Apazidis [20].

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2. Problem formulation

Consider natural convection flow of a two-phase dusty fluid moving along the vertical wavy frustum of a cone under the influence of thermal radiation. The shape of the wavy surface, $\hat{\sigma}(\hat{x})$, is arbitrary, but our detailed numerical work will assume that the surface exhibits sinusoidal deformations. Thus, the wavy frustum of the vertical cone is described by

$$\hat{y}_w = \hat{\sigma}(\hat{x}) = \hat{a}\sin\left(\frac{\pi(\hat{x}-\hat{x}_0)}{L}\right) \tag{1}$$

where $\hat{a}$ is the dimensional amplitude of the wavy surface, $\hat{x}_0$ is the slant height of the lower end of the cone, and L is the characteristic length associated with the uneven surface (also known as half of the wavelength of the uneven surface). We have considered dusty fluid, which is originally at rest along a vertical heated wavy frustum of the cone. Initially, the system is having a uniform temperature T_∞. Suddenly, the surface of the cone $\hat{y}=0$ is heated to a temperature $T+\Delta T$ and natural convection starts due to this. Fig. 1 shows the geometry of the wavy surface and the two-dimensional Cartesian system. Under the assumptions of the dusty fluid flow given in (see Ref. [3,8]), the governing system of equations is developed as follows:

For the gas phase:

$$\frac{\partial(\hat{r}\hat{u})}{\partial\hat{x}} + \frac{\partial(\hat{r}\hat{v})}{\partial\hat{y}} = 0 \tag{2}$$

$$\rho\left(\hat{u}\frac{\partial\hat{u}}{\partial\hat{x}} + \hat{v}\frac{\partial\hat{u}}{\partial\hat{y}}\right) = -\frac{\partial\hat{p}}{\partial\hat{x}} + \mu\nabla^2\hat{u} + \rho g\beta(T-T_\infty)\cos\phi + \frac{\rho_p}{\tau_m}(\hat{u}_p-\hat{u}) \tag{3}$$

$$\rho\left(\hat{u}\frac{\partial\hat{v}}{\partial\hat{x}} + \hat{v}\frac{\partial\hat{v}}{\partial\hat{y}}\right) = -\frac{\partial\hat{p}}{\partial\hat{y}} + \mu\nabla^2\hat{v} - \rho g\beta(T-T_\infty)\sin\phi + \frac{\rho_p}{\tau_m}(\hat{v}_p-\hat{v}) \tag{4}$$

$$\rho c_p\left(\hat{u}\frac{\partial T}{\partial\hat{x}} + \hat{v}\frac{\partial T}{\partial\hat{y}}\right) = \kappa\nabla^2 T - \nabla.\vec{q}_r + \frac{\rho_p c_s}{\tau_T}(T_p-T) \tag{5}$$

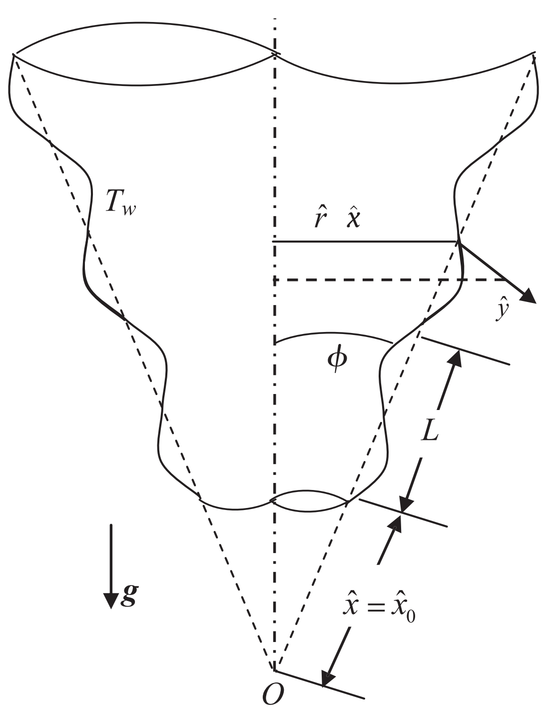

Fig. 1. Schematic of the problem.

For the particle phase:

$$\frac{\partial(\hat{r}\hat{u}_p)}{\partial\hat{x}} + \frac{\partial(\hat{r}\hat{v}_p)}{\partial\hat{y}} = 0 \tag{6}$$

$$\rho_p\left(\hat{u}_p\frac{\partial\hat{u}_p}{\partial\hat{x}} + \hat{v}_p\frac{\partial\hat{u}_p}{\partial\hat{y}}\right) = -\frac{\partial\hat{p}_p}{\partial\hat{x}} - \frac{\rho_p}{\tau_m}(\hat{u}_p-\hat{u}) \tag{7}$$

$$\rho_p\left(\hat{u}_p\frac{\partial\hat{v}_p}{\partial\hat{x}} + \hat{v}_p\frac{\partial\hat{v}_p}{\partial\hat{y}}\right) = -\frac{\partial\hat{p}_p}{\partial\hat{y}} - \frac{\rho_p}{\tau_m}(\hat{v}_p-\hat{v}) \tag{8}$$

$$\rho_p c_s\left(\hat{u}_p\frac{\partial T_p}{\partial\hat{x}} + \hat{v}_p\frac{\partial T_p}{\partial\hat{y}}\right) = -\frac{\rho_p c_s}{\tau_T}(T_p-T) \tag{9}$$

where $(\hat{u},\hat{v})$, T, $\hat{p}$, ρ, c_p, β, κ, and μ are the velocity vector in the $(\hat{x},\hat{y})$ directions, temperature, pressure, density, specific heat at constant pressure, volumetric expansion coefficient, thermal conductivity, and coefficient of viscosity of the fluid/carrier phase, respectively. Similarly, $(\hat{u}_p,\hat{v}_p)$, T_p, $\hat{p}_p$, ρ_p, and c_s correspond to the velocity vector, temperature, pressure, density, and specific heat for the particle phase, respectively. $\vec{g}=(g\cos\phi, g\sin\phi)$ is the gravitational acceleration along the $(\hat{x},\hat{y})$ directions, respectively; ϕ is the half angle; and $\hat{r}=(\hat{x}+\hat{x}_\circ)\sin\phi$ the local radius of the frustum of a cone. Likewise, τ_m (τ_T) is the momentum relaxation time (thermal relaxation time) during which the velocity (temperature) of the particle phase relative to the fluid is reduced to $1/e$ times its initial value. Here, the carrier fluid is assumed to be gray, emitting and absorbing, but non-scattering medium. The fundamental equations stated above are to be solved under appropriate boundary conditions to determine the flow fields of the fluid and the dust particles. Therefore, boundary conditions for the gas phase are

$$\begin{aligned}\hat{u}(\hat{x},\hat{y}_w) = \hat{v}(\hat{x},\hat{y}_w) = T(\hat{x},\hat{y}_w)-T_w = 0\\ \hat{u}(\hat{x},\infty) = T(\hat{x},\infty)-T_\infty = 0\end{aligned} \tag{10}$$

Boundary conditions for the particle phase are

$$\begin{aligned}\hat{u}_p(\hat{x},\hat{y}_w) = \hat{v}_p(\hat{x},\hat{y}_w) = T_p(\hat{x},\hat{y}_w)-T_w = 0\\ \hat{u}_p(\hat{x},\infty) = T_p(\hat{x},\infty)-T_\infty = 0\end{aligned} \tag{11}$$

T_w is the constant temperature of the heated cone, which is higher than the ambient fluid temperature T_∞. In Eq. (5), the term $\vec{q}_r$ is the radiative heat flux term in the $\hat{y}$ direction, and due to the Rosseland diffusion approximation, it can be written as

$$\vec{q}_r = -\frac{4\sigma^*}{3\kappa(\alpha_r+\sigma_s)}\nabla T^4 \tag{12}$$

where σ^*, α_r, and σ_s are the Stephan–Boltzmann constant, the Rosseland mean extinction coefficient, and the scattering coefficient, respectively. For large Gr, the system of Eqs. (2)–(11) can be transformed into boundary layer equations as follows:

For the gas phase:

$$\frac{\partial(ru)}{\partial x} + \frac{\partial(rv)}{\partial y} = 0 \tag{13}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{\partial p}{\partial x} + \sigma_x Gr^{1/4}\frac{\partial p}{\partial y} + (1+\sigma_x^2)\frac{\partial^2 u}{\partial y^2} + \theta + D_\rho\alpha_d(u_p-u) \tag{14}$$

$$\sigma_x\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) + \sigma_{xx}u^2 = -Gr^{1/4}\frac{\partial p}{\partial y} + \sigma_x(1+\sigma_x^2)\frac{\partial^2 u}{\partial y^2} - \tan\phi\theta + \sigma_x D_\rho\alpha_d(u_p-u) \tag{15}$$