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Rayleigh–Bénard convection in non-Newtonian Carreau fluids with arbitrary conducting boundaries☆

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ABSTRACT

The objective of the present work is to investigate the Rayleigh–Bénard convection in non-Newtonian fluids with arbitrary conducting boundaries. A linear and weakly nonlinear analysis is performed. The rheological behavior of the fluid is described by the Carreau model. As a first step, the critical Rayleigh number and wavenumber for the onset of convection are computed as a function of the ratios ξ^b and ξ^t of the thermal conductivities of the bottom and top slabs to that of the fluid. In the second step, the preferred convection pattern is determined using an amplitude equation approach. The stability of rolls and squares is investigated as a function of (ξ^b, ξ^t) and the rheological parameters. The bounded region of (ξ^b, ξ^t) space where squares are stable decreases with increasing shear-thinning effects. This is related to the fact that shear-thinning effects increase the nonlinear interactions between sets of rolls that constitute the square patterns [1]. For a significant deviation from the critical conditions, the nonlinear convection terms and nonlinear viscous terms become stronger, reducing overall the stability domain of squares. The largest Nusselt number, *Nu*, is obtained for perfectly conducting boundaries. For a given (ξ^b, ξ^t), the stable solution yields the largest Nusselt number. The enhancement of heat transfer due to shear-thinning effects is significantly reduced for poorly heat conducting plates.

1. Introduction

The problem of Rayleigh–Bénard convection (RBC) in Newtonian and non-Newtonian fluids layer heated from below and cooled from above remains one of the classical problems of fluid dynamics and heat transfer. In spite of intensive studies made in the past and extensive research work undertaken so far to understand the competition between convective structures (rolls, squares and hexagons) which are often influenced by the boundary conditions (see Holmedal et al. [2]; Clever and Busse [3]) and other parameters such as temperature-dependence of viscosity (see White [4]; Palm [5]; Richter [6]; Olivier and Booker [7]; Busse and Frick [8]; Jenkins [9]), there are still many outstanding issues that need to be answered.

Maybe one of the most important question to be addressed is the effect of conductive horizontal plates on the heat transfer and the convection patterns. For instance, in geophysical problems and particularly in the context of the Earth's mantle convection, continents and oceans impose different thermal boundary conditions at the top of the mantle: continents act as insulators while a fixed temperature is imposed by oceans. These different thermal boundary conditions affect the convective flow and the heat transport in the Earth's mantle [10].

Actually, in most numerical investigations of RBC, the plates are assumed to be infinitely heat conducting, and a fixed temperature at the boundaries is imposed, while in engineering and geophysical problems as well as in laboratory experiments the boundaries have a finite conductivity. This may lead to a discrepancy between the experimental and the numerical/theoretical results. The ratios ξ^b and ξ^t between the thermal conductivities of the bottom and top slabs and that of the fluid may have a significant effect and must be taken into account as additional parameters [11].

The influence of the thermal conductivity of the boundaries on Rayleigh–Bénard convection was first investigated in the Newtonian case by Busse and Riahi [12] using a weakly nonlinear analysis. They considered the situation where $\xi^b = \xi^t = \xi \ll 1$ and found that the wavelength of convection flow becomes very large in comparison

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with the height of the layer and only square patterns are stable. This result was confirmed and extended to the fully nonlinear problem by Proctor [13] using a 'shallow water theory'. Afterwards, Jenkins and Proctor [14] determined the critical value of the thermal conductivities ratios $\xi^b = \xi^t = \xi_c$ at which the preferred planform changes from square cell to roll. For $Pr > 10$, they found that the preferred planform is rolls when $\xi > 1$, and squares when $\xi < 1$. Le Gal et al. [15] carried out experiments to study Rayleigh–Bénard convection in silicone oil confined between two glass plates. So that $\xi^b = \xi^t = \xi = 7$. Near the threshold of convective instability, at $\epsilon < 0.021$, where ϵ is the relative distance from the onset of instability, they observed cells of square planforms. But when $0.024 < \epsilon < 0.057$, the amplitude of two mutually perpendicular roll sets underwent periodic oscillations in antiphase with another; as ϵ was increased and convection became more intense, one set became predominant and then a unique steady-state roll set was established. This experiment was subsequently modified by Le Gal and Croquette [16] : glass was replaced by plexiglass and water was used as the working fluid, so that $\xi = 0.4$. In contrast to the preceding experiment, squares were observed in a wide range of ϵ values without any signs of destabilization. The authors think that in the first case, the silicone oil behaves as a mixture and the observed features were governed by the thermophoresis.

Although extensive studies have been devoted to understand the influence of the thermal boundary conditions on the Rayleigh–Bénard convection in Newtonian fluids, only a limited number of works have dealt with complex fluids. In comparison with the Newtonian system, the nonlinearity of the rheological law introduces an additional coupling in the velocity component. Recently, Bouteraa and Nouar [17] have investigated the influence of shear-thinning effects on the convection in a horizontal layer of a shear-thinning fluid between two horizontal symmetric plates of finite thermal conductivity. The rheological behavior of the fluid is described by the Carreau model. The authors found that: (i) the characteristic time of instability τ_0 increases significantly when $\xi < 1$, (ii) the critical value of the shear-thinning degree α_c above which the bifurcation becomes subcritical increases with decreasing ξ, and (iii) the critical value ξ_c at which the planform changes from square-cell solution ($\xi < \xi_c$) to two-dimensional roll solution ($\xi > \xi_c$) decreases with increasing shear-thinning effects.

In some experimental situations, $\xi^t \neq \xi^b$. For Newtonian fluids, Riahi [11,18] has studied this problem and demonstrated, using a linear stability analysis of stationary flows the enormous influence of thermal boundary conditions (when $\xi^b \neq \xi^t$) on the competition between the convection patterns. He found that squares are stable when rolls are unstable and vice versa, and always hexagonal patterns are unstable. No hysteresis effect is found. In addition, Riahi [18] has also shown that square planforms are preferred in a bounded region Ω in the (ξ^b, ξ^t)-space coordinate system and rolls are favored only outside Ω. When $Pr < 0.025$, the region Ω is quite small and disappears as $Pr = 0$. However, for $Pr > 7$, Ω is largest and nearly independent of Pr. Using nonlinear developments, Clever and Busse [3,19] demonstrated in the case of stress-free nearly insulating top plate and highly conducting no-slip lower plate, that two-dimensional rolls are stable near the onset, but become unstable at higher Rayleigh number and are replaced by which is called hexaroll convection.

From experimental point of view, Darbouli et al. [20] have investigated Rayleigh–Bénard convection for viscoplastic fluids confined in a cylindrical cell. They used two different horizontal plates of finite thermal conductivity. The bottom and upper walls are made respectively of copper alloy and glass. They used distilled water as Newtonian fluid to validate their experimental setup and an aqueous solution of Carbopol 940 as viscoplastic fluid. In these situations, the ratios ξ^t and ξ^b are estimated to $\xi^t = 2$ and $\xi^b = 201.6$ for both fluids (authors estimated that the solution of Carbopol 940 has the same thermal conductivity than water). Hence, it is no longer possible to rely on the assumption that the plates are held at fixed and uniform temperatures, which corresponds to plates with infinite thermal conductivity.

The purpose of the present work is to study the influence of arbitrary thermal-conducting top and bottom boundaries on nonlinear processes of Rayleigh–Bénard convection, and to see the influence of the shear-thinning effect on the preferred flow pattern. The finite conductivity of the slabs remains one explanation for differences between results obtained in experiments and numerical investigations. We hope that our findings will shed new light on the interpretation of the results obtained by Darbouli et al. [20] although the fluid used is not only shear-thinning but has also a yield stress.

2. Physical and mathematical model

2.1. General equations and parameters

We consider a horizontal layer of a shear-thinning fluid of height $\hat{d}$ confined between two horizontal plates that are infinite in extent and which have a thickness $\Lambda\hat{d}$, where Λ is of order unity. The outer surface of the bottom and top plates are kept at constant temperatures respectively $\hat{T}_0 + \Delta\hat{T}/2$ and $\hat{T}_0 - \Delta\hat{T}/2$, with $\Delta\hat{T} > 0$. The fluid has density $\hat{\rho}$, thermal conductivity $\hat{k}$, thermal coefficient expansion (at constant pressure) $\hat{\beta}$ and viscosity $\hat{\mu}_0$ at zero shear rate. The top slab has a thermal conductivity $\hat{k}_p^t$ and a thermal diffusivity $\hat{\kappa}_p^t$. The corresponding quantities for the bottom slab are denoted $\hat{k}_p^b$ and $\hat{\kappa}_p^b$. Here and in what follows, (t) and (b) refer to the top and bottom and the quantities with hat $(\hat{.})$ are dimensional. Because of the thermal expansion, the temperature difference between the two plates, induces a vertical density stratification. Heavy cold fluid is above a light warm fluid. For small $\Delta\hat{T}$, the fluid remains motionless and the heat is transferred by conduction, with a linear temperature profile across the fluid layer.

In the fluid, $0 < \hat{z} < \hat{d}$, the hydrostatic solution and the temperature profile are:

$$\frac{d\hat{P}}{d\hat{z}} = -\hat{\rho}_0\hat{g}\left[1 - \hat{\beta}\left(\hat{T} - \hat{T}_0\right)\right] \quad \text{and}$$
$$\hat{T}_{cond} = \hat{T}_0 + \frac{\Delta\hat{T}}{1 + \Lambda/\xi^{(b)} + \Lambda/\xi^{(t)}}\left[\frac{1}{2} - \frac{\hat{z}}{\hat{d}}\right], \tag{1}$$

where, $\hat{g}$ is the acceleration due to gravity. Here, the z-axis is directed upwards, with the origin located at the bottom plate. The reference temperature $\hat{T}_0$ is the temperature in the middle of the fluid layer and $\hat{\rho}_0$ is the fluid density at $\hat{T}_0$. Here, $\hat{T}_0 = \hat{T}_1 - \left(1/2 + \Lambda/\xi^b\right)\Delta\hat{T}_f$, where $\hat{T}_1$ is the temperature on the outer surface of the bottom plate and $\Delta\hat{T}_f$ the temperature difference between the top and the bottom of the fluid layer: $\Delta\hat{T}_f = \Delta\hat{T}/(1 + \Lambda/\xi^t + \Lambda/\xi^{(b)})$.

The temperature profile in the top and bottom plates are:

$$\hat{T}_{cond} = \hat{T}_0 + \frac{\Delta\hat{T}}{\xi^t + \Lambda\left(1 + \xi^t/\xi^b\right)}\left[1 + \frac{\Lambda}{2}\left(1 - \frac{\xi^t}{\xi^b}\right) - \frac{1}{2}\xi^t - \frac{\hat{z}}{\hat{d}}\right],$$
$$\hat{d} \le \hat{z} \le (1 + \Lambda)\hat{d} \tag{2}$$

and

$$\hat{T}_{cond} = \hat{T}_0 + \frac{\Delta\hat{T}}{\xi^b + \Lambda\left(1 + \xi^b/\xi^t\right)}\left[\frac{1}{2}\xi^b - \frac{\Lambda}{2}\left(1 - \xi^b/\xi^t\right) - \frac{\hat{z}}{\hat{d}}\right],$$
$$-\Lambda\hat{d} \le \hat{z} \le 0. \tag{3}$$

When the bottom and top plates are poor thermal conductors, a large part of $\Delta\hat{T}$ occurs across the plates, and remains only a small