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Natural transition in natural convection boundary layers☆

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ABSTRACT

The 'natural transition' of a natural convection boundary layer adjacent to an isothermally heated vertical surface is investigated by means of three-dimensional direct numerical simulation (DNS). In order to trigger 'natural transition' numerically, spatially and temporally random perturbations are introduced into the upstream boundary layer. The propagation of the random perturbations in the streamwise direction is observed. It is found that there exist two competing wavenumbers of spanwise vortical structures, one large and the other small. The large wavenumber dominates in the upstream boundary layer, whereas the small wavenumber dominates in the downstream boundary layer. The streamwise evolution of the mean (time-averaged) streamwise vorticity observed at planes perpendicular to the heated surface in general reveals two- and three-layer longitudinal roll structures. Nonlinear processes in a transitioning natural convection boundary layer are also analysed using Bicoherence method. The transition route and mechanism are discussed based on the power spectra and Bicoherence spectra of the temperature time series obtained in the boundary layer. A spectrum filling process during the 'natural transition' to turbulence is also observed, which is qualitatively similar to that observed in Blasius boundary layers.

1. Introduction

The complete mechanism of laminar-turbulent transition of boundary-layer flows has been a mystery for decades in the fluid mechanics community, which has stimulated extensive and intensive research in this area. The research on the transition of boundary layers has been directed to two main streams: '*natural transition*' and '*controlled transition*'. 'Natural transition' refers to the transition subject to arbitrary environmental disturbances, whereas 'controlled transition' refers to the transition subject to predefined disturbances with known frequency, amplitude and wavelength.

One of the pioneering experimental works with regard to 'controlled transition' in Blasius boundary layers was reported by Klebanoff et al. [1]. The transition revealed in [1] was classified as a K-type 'controlled' transition, which was excited by superimposed Tollmien–Schlichting (TS) and oblique waves of the same frequency. The PIV experiment in [2] and the direct numerical simulation in [3] demonstrated that the K-type transition is characterized by aligned ^-shaped structures in the boundary layer. Another typical 'controlled transition' is the H-type transition, which was studied by Kachanov et al. [4], Kachanov and Levchenko [5], Craik [6], Herbert [7], Berlin et al. [2], Sayadi et al. [3] and others. The H-type transition is characterized by staggered ^-shaped structures in the boundary layer. The reviews in [8,9] discussed the characteristics of both types of transition in detail.

Existing research on 'natural transition' in natural convection boundary layers is mainly concerned with the two-dimensional linear instability of the transitional process, rather than the three-dimensional transitional process. For the two-dimensional linear instability of the transitional process, many linear stability analyses (e.g. [10–12]), experimental investigations (e.g. [13–16]) and direct stability analyses (e.g. [17–25]) have been devoted to this topic. Among the existing studies, Zhao et al. [25] has documented comprehensive instability characteristics of natural convection boundary layers.

With regard to 'controlled transition' in natural convection boundary layers, the pioneering work was conducted by Jaluria and Gebhart [26]. The studied and observed transition in [26] was in fact a K-type transition. A principal inference in [26] was the existence of a double longitudinal vortex system, which was recognized as an important mechanism responsible for the distortion of the base velocity profile. Audunson and Gebhart [27] later found that the double longitudinal vortex system was caused by the finite perturbation amplitude and the interaction between the spanwise and streamwise disturbances.

In the present study, the three-dimensional 'natural transition' of natural convection boundary layers is studied by means of direct numerical simulation for the first time. Spatially and temporally random perturbations, which are analogous to non-homogenous environmental disturbances covering a broad band of competing frequencies and wavelengths, are introduced into the upstream boundary layer. The evolution of the random perturbations in the streamwise direction is

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followed. Firstly the streamwise vorticities, the corresponding wavenumbers and the streamwise evolution of the wavenumbers in the transitioning boundary layer are examined at a Rayleigh number (defined in Eq. (7) below) $Ra = 3.5 \times 10^9$. Subsequently, the transition route and the mechanism for the transition are investigated at a higher Rayleigh number of $Ra = 1.4 \times 10^{10}$ by means of spectral analysis and Bicoherence analysis.

2. Mathematical formulation

2.1. Problem description

Under consideration is a Newtonian three-dimensional (3D) natural convection boundary-layer flow induced by an isothermally heated vertical surface (refer to the schematic shown in Fig. 1). A domain size of $L \times H \times W$ is used for the direct numerical simulation. Here, H is the height of the isothermally heated surface. The determination of the dimension L is based on the thickness scale of the viscous boundary layer, which is $\delta_v = Pr^{1/2}H/Ra^{1/4}$ [28], to ensure that the far-field boundary condition is satisfied. In the present simulations, L is set at $L = 0.14H$, which is about 13 times the predicted thickness of the viscous boundary layer for $Ra = 3.5 \times 10^9$ and 18 times for $Ra = 1.4 \times 10^{10}$. To minimize the effects of the end boundaries, the computational domain is extended at the top and bottom by $H_t = H_b = 0.1H$ respectively. No penalty method is applied in the extended regions and the results in the present study are obtained within the basic domain $L \times H \times W$ only. Similar strategies have been employed in [29,30].

The determination of the dimension W is based on the wavelength of the streamwise vorticity occurring in the boundary layer undergoing 'natural transition'. In general, the effects of the dimension W on the characteristics of the transitioning flow will reduce as the dimension increases, as reported by Lei et al. [31] in a relevant study. In the present study of the 3D structures of the 'natural transition', the spanwise dimension W of the computational domain is chosen to be at least 2.5 times the spanwise wavelength of the streamwise vorticity based on the wavelengths obtained from numerical tests using various values of W. The effects of the spanwise dimension W on the spatial and temporal wavenumbers will be discussed later.

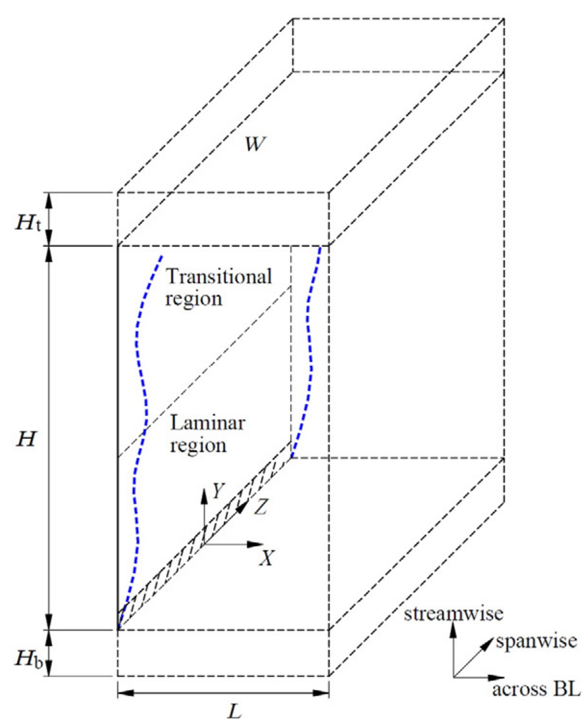

Fig. 1. Schematic of the computational domain, with the shaded strip near the leading edge of the heated surface showing the region where perturbations are introduced into the boundary layer.

2.2. Governing equations

The flow under consideration is described by the three-dimensional, incompressible Navier–Stokes and energy equations. The non-dimensional forms of these governing equations, under the Boussinesq approximation, are expressed as follows:

$$u_x + v_y + w_z = 0, \tag{1}$$

$$u_t + uu_x + vu_y + wu_z = -p_x + \nabla^2 u, \tag{2}$$

$$v_t + uv_x + vv_y + wv_z = -p_y + \nabla^2 v + \theta \cdot Ra/Pr, \tag{3}$$

$$w_t + uw_x + vw_y + ww_z = -p_z + \nabla^2 w, \tag{4}$$

$$\theta_t + u\theta_x + v\theta_y + w\theta_z = \nabla^2 \theta / Pr, \tag{5}$$

where u, v and w are the velocity components in the x, y and z directions, respectively; p, t and θ are the pressure, time and temperature; and Ra and Pr are the Rayleigh and Prandtl numbers. The dimensionless quantities in the governing Eqs. (1)–(5) are obtained as follows:

$$\left. \begin{array}{lll} u = \dfrac{U}{\nu H^{-1}}, & v = \dfrac{V}{\nu H^{-1}}, & w = \dfrac{W}{\nu H^{-1}}, \\ x = \dfrac{X}{H}, & y = \dfrac{Y}{H}, & z = \dfrac{Z}{H}, \\ t = \dfrac{\tau}{H^2 \nu^{-1}}, & p = \dfrac{P}{\rho \nu^2 H^{-2}}, & \theta = \dfrac{T - T_0}{T_h - T_0}, \end{array} \right\} \tag{6}$$

in which U, V, W, X, Y, Z, τ, P and T are the corresponding dimensional quantities.

The two control parameters of the boundary-layer flow under consideration are the Rayleigh number Ra and the Prandtl number Pr, defined as:

$$Ra = \frac{g\beta\Delta T H^3}{v\kappa}, \quad Pr = \frac{\nu}{\kappa} \tag{7}$$

where g and ΔT are the gravitational acceleration and the temperature difference between the isothermal wall and the ambient respectively; β, ν and κ are the thermal expansion coefficient, kinematic viscosity and thermal diffusivity of the working fluid at the reference (ambient) temperature T_0.

Initially, the fluid in the computational domain is stationary and isothermal at the non-dimensional temperature $\theta = 0$. For $t > 0$, the following boundary conditions are prescribed:

$$\left. \begin{array}{l} u = v = w = 0, \quad \theta = 1 + \xi \ \text{at}\ x = 0, \quad 0 \le y < 0.02, \quad -\sigma \le z \le \sigma, \\ u = v = w = 0, \quad \theta = 1 \ \text{at}\ x = 0, \quad 0.02 \le y \le 1.1, \quad -\sigma \le z \le \sigma, \\ u = v = w = 0, \quad \theta_x = 0 \ \text{at}\ x = 0, \quad -0.1 \le y < 0, \quad -\sigma \le z \le \sigma, \\ u = v = w = 0, \quad \theta_y = 0 \ \text{at}\ y = -0.1, \quad 0 < x \le 0.14, \quad -\sigma \le z \le \sigma, \\ u_y = v_y = w_y = 0, \quad \theta_y = 0 \ \text{at}\ y = 1.1, \quad 0 < x \le 0.14, \quad -\sigma \le z \le \sigma, \\ u_x = v_x = w_x = 0, \quad \theta_x = 0 \ \text{at}\ x = 0.14, \quad -0.1 < y < 1.1, \quad -\sigma \le z \le \sigma, \\ \Gamma(x, y, -\sigma, t) = \Gamma(x, y, \sigma, t). \end{array} \right\} \tag{8}$$

where ξ is a temperature perturbation prescribed on the boundary and $\sigma = W/2H$ represents the normalized half spanwise width of the computational domain. $\Gamma(x, y, \pm\sigma)$ represents an arbitrary flow quantity on the xy-planes at $z = \pm\sigma$. The prescription of $\Gamma(x, y, -\sigma, t) = \Gamma(x, y, \sigma, t)$ establishes a periodic boundary condition in the spanwise direction.