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On the effects of internal heat sources upon forced convection in porous channels with asymmetric thick walls☆

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ABSTRACT

Thermal behaviour of a porous channel with thick, solid walls featuring uneven wall thicknesses and asymmetric external thermal boundary conditions is analysed theoretically. The system is under forced convection and the fluid and solid phases in this configuration include internal heat sources with varying strengths. Two types of asymmetric boundary conditions are considered. These include constant but different prescribed temperatures on the upper and lower solid walls and a combination of constant heat flux and convective boundary conditions on the two sides of the channel. The Darcy–Brinkman model of momentum transport and the two-equation energy model are utilised to develop analytical solutions for the temperature fields and Nusselt number. A comprehensive parametric study is, subsequently, conducted. The results clearly show the pronounced effect of the internal heat sources upon the Nusselt number and temperature fields of the system. In particular, the existence of these source terms intensifies the occurrence of a bifurcation phenomenon in the temperature fields. In keeping with the recent literature, it is demonstrated that the inclusion of internal heat sources leads to deviations from the local thermal equilibrium. Nonetheless, the results imply that the extent of these deviations depends on the thermal boundary conditions and also the specific phase in which heat is generated or consumed.

1. Introduction

The problem of convective heat transfer in porous media has received increasing attention over the last few decades [1,2]. The growing significance of this topic can be attributed to a few reasons. These include the direct applications of transport in porous media in many conventional engineering fields such as thermal systems, chemical reactors and oil and gas reservoirs [3,4]. The subject has further found new applications in emerging fields such as biotechnology and biomedical engineering [5]. The sensitivity of these systems and the recent emphasis on improving the energy efficiencies have greatly signified the need for superior thermal models. Central to achieving this goal is the consideration of more realistic situations and, therefore, releasing the simplifying assumptions [6].

A survey of the literature reveals that a large fraction of the existing theoretical analyses in the field of forced convection in porous media includes some common simplifying assumptions [1–3]. Consideration of local thermodynamic equilibrium, axisymmetric configurations and ignoring the internal heat sources are amongst these assumptions. Application of non-equilibrium thermodynamics has manifested itself, mostly, in the utilisation of the local thermal non-equilibrium (LTNE) or two-energy equation method [7–9]. Over the last two decades, LTNE has been applied to various flow conduits, which were fully [10–12] or partially [13–16] filled by porous materials. This has resulted in improved predictions of the temperature distribution of the individual phases in porous media. Nonetheless, the unresolved problem of thermal boundary conditions on the porous-solid and porous-fluid interfaces continues to challenge this approach [17,19,20]. Asymmetric configurations have been considered in a number of works. These often include asymmetric flow conduits partially filled by a porous insert such that the symmetry-breaking element is the location of the porous insert, see for example [21,22]. The asymmetric configurations with thick solid walls have received much less attention. The latter configuration is, generally, a lesser explored setting in the modelling of thermal systems. Recently, Ibanez et al. [23] considered the problem of heat and fluid flow in a clear micro-channel featuring thick walls. These authors [23] assumed constant thermal conductivity for the solid walls and developed analytical solutions for the momentum and energy equations. Their results clearly demonstrated the significance of thick walls in the thermal behaviour of the system [23]. The growing importance of the thermal analysis of purely conductive or conductive-convective components is also reflected by the recent research interests in this subject [24–26].

The internal heat sources in porous media under local thermal equilibrium (LTE) conditions have been included in some studies [27–29].

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Nomenclature

Bi	Biot number defined in Eq. (11a-t)
Da	Darcy number
h	Convection heat transfer (Case two), $W \cdot m^{-2} \cdot K^{-1}$
h_3	Height of the channel, m
k_1	Reference thermal conductivity for lower solid material, $W \cdot m^{-1} \cdot K^{-1}$
k_2	Reference thermal conductivity for upper solid material, $W \cdot m^{-1} \cdot K^{-1}$
k_{ef}	Effective thermal conductivity of the fluid phase of the porous medium, $W \cdot m^{-1} \cdot K^{-1}$
k_{es}	Effective thermal conductivity of the solid phase of the porous medium, $W \cdot m^{-1} \cdot K^{-1}$
k_{e1}	Ratio of porous medium thermal conductivity to lower solid material thermal conductivity
k_{e2}	Ratio of porous medium thermal conductivity to upper solid material thermal conductivity
Nc	Dimensionless convection heat transfer (Case two)
Q_1	Dimensionless volumetric internal heat generation rate for the lower solid material
Q_2	Dimensionless volumetric internal heat generation rate for the upper solid material
Q_H	Dimensionless heat flux boundary condition (Case two)
w_s	Dimensionless volumetric internal heat generation rate for the solid phase of the porous medium
w_f	Dimensionless volumetric internal heat generation rate for the fluid phase of the porous medium
s_s	Volumetric internal heat generation rate for the solid phase of the porous medium
s_f	Volumetric internal heat generation rate for the fluid phase of the porous medium
$\dot{q}_1$	Volumetric internal heat generation rate for the lower solid material, $W \cdot m^{-3}$
$\dot{q}_2$	Volumetric internal heat generation rate for the upper solid material, $W \cdot m^{-3}$
q_H	Heat flux boundary condition (Case two), $W \cdot m^{-2}$
T	Temperature, K
T_1	Temperature of the lower solid material, K
T_2	Temperature of the upper solid material, K
T_C	Outer temperature of the upper solid material, K
T_H	Inner temperature of the lower solid material, K
T_f	Temperature of the fluid phase of the porous medium, K
T_s	Temperature of the solid phase of the porous medium, K
U_p	Dimensionless velocity
u_p	Velocity of the fluid in porous medium, $m \cdot s^{-1}$
γ_1	Dimensionless wall thickness defined in Eq. (11a-t)
γ_2	Dimensionless wall thickness defined in Eq. (11a-t)

Greek symbols

κ	Permeability, m^2
μ_{eff}	Dynamic viscosity of porous medium, $Kg \cdot s^{-1} \cdot m^{-1}$
μ_f	Dynamic viscosity of the base fluid, $Kg \cdot s^{-1} \cdot m^{-1}$
θ	Dimensionless temperature
θ_1	Dimensionless temperature of the lower solid material
θ_2	Dimensionless temperature of the upper solid material
θ_f	Dimensionless temperature of the fluid phase of the porous medium
θ_s	Dimensionless temperature of the solid phase of the porous medium
θ_H	Dimensionless temperature at outer side of the lower wall

These works mostly concentrated on heat generation by viscous dissipation. An exception to this is the work of Chen et al. [27], which considered uniform internal heat generations under local thermal equilibrium. Examples of LTNE analyses with internal heat sources are much less frequent and mostly limited to the recent studies. In a theoretical work, Yang and Vafai [30] investigated a fully filled porous channel under LTNE condition, which also featured internal heat generations. They considered two different porous-solid thermal interface models and developed closed form analytical solutions for the temperature fields and Nusselt number [30]. Yang and Vafai demonstrated that internal heat generation could cause significant deviations from the local equilibrium condition [30]. Most recently, this work was extended to the partially filled porous channels by Karimi et al. [31] and Torabi et al. [32]. Uniform exothermic and endothermic processes were assumed to generate or consume thermal energy in the fluid and solid phases. In keeping with the earlier work of Yang and Vafai [30], these authors [31,32] showed the strong effects of internal heat sources on the thermal behaviour of the system. They also demonstrated the possibility of occurring heat flux and temperature bifurcations [31,32]. In particular, these studies confirmed the necessity of taking the non-equilibrium approach in the analysis of problems which involve internal heat sources [31,32].

In reality, there are many thermal problems in porous media which are not under local thermodynamic equilibrium, include asymmetric configurations and involve exothermic or endothermic processes. Chemical and nuclear reactors are the typical examples of this class of problems, while biological systems are another application field, in which metabolism provides the internal source of heat [33]. Biological systems are usually asymmetric and can be subject to different thermal boundary conditions [34]. All these examples accommodate exothermic or endothermic reactions in the fluid and solid phases of the porous medium. As a result, they are likely to operate far from the local thermal equilibrium condition [35]. Further, most reactors include high pressures which necessitate using thick walls. In practice, the thickness of the wall may vary at different points resulting in an asymmetric configuration. Furthermore, reactors may be subject to various types of waves (e.g. infrared, beta and gamma waves) [36]. Absorption of these waves forms a source of thermal energy in the walls of the system. Similarly, in biological applications, the region of interest is normally surrounded by other heat generating tissues. Asymmetric porous systems with thick walls have been most recently analysed by Torabi and Zhang [37]. These authors considered magneto-hydrodynamic effects and solved the governing equations analytically to find the velocity, temperature and entropy generation rates [37]. However, their work did not include internal heat sources in the porous medium [37].

The preceding review of the literatures reveals that the thermal analysis of porous media with asymmetric configuration and internal heat sources sets a challenge that has not been previously met. In particular, the influences of internal heat sources upon the temperature and heat transfer rates are currently completely unknown. The aim of this work is to address this issue through a series of theoretical analyses.

2. Theoretical methods

2.1. Problem configuration and assumptions

Fig. 1 shows the schematic view of the problem under investigation. The channel is fully filled by a porous material and includes thick walls with constant, but distinctive, thermal conductivities as well as constant and uniform, but dissimilar, internal heat generations. The internal heat generation within the solid walls could be, for example, the result of the absorption of gamma rays in the solid walls [38,39]. Two sets of boundary conditions are considered in this problem. In Case one (Fig. 1a), it is assumed that the upper and lower surfaces are subject to constant but different temperatures. Case two (Fig. 1b) includes a constant heat flux on the lower wall and a convective boundary condition on the upper wall.