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A note on the stagnation-point flow over a permeable shrinking sheet with slip effects[☆]

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ABSTRACT

A detailed study of the problem of the boundary-layer flow on a shrinking permeable surface near a forward stagnation point with an outer flow $u_\infty \propto x^m$, a tangential wall velocity $u_w \propto x^m$, and velocity slip on the surface considered previously by Fauzi et al. [1] for $m = 1$ (stagnation point flow) is presented. Further numerical results are obtained and the asymptotic behaviour of the flow under various conditions of the governing parameters is described. Four cases of the problem are considered, namely an impermeable fixed wall, an impermeable moving wall, a permeable fixed wall and a permeable moving wall. For the case of an impermeable fixed wall, it is found that there is a critical value β_c of $\beta = 2m/(m+1)$ dependent on the velocity slip parameter A, and that this critical value approaches a finite limit as A increases. For the case of impermeable moving wall, the critical value is negative, decreasing as A is increased. Asymptotic solutions for both strong suction and strong blowing are obtained for the permeable fixed wall. For the case of permeable moving wall, the critical values λ_c of the parameter λ, the ratio of the wall velocity to the outer flow, found by Fauzi et al. [1] are completed and plotted against the governing suction parameter S. It is seen that λ_c becomes large as suction is increased.

1. Introduction

In a recent paper Fauzi et al. [1] considered the steady boundary-layer flow on a shrinking permeable surface with an outer flow $u_\infty \propto x^m$ and a tangential wall velocity $u_w \propto x^m$, though they were mostly concerned with a forward stagnation-point flow, i.e. taking $m = 1$. They also included the effect of velocity slip on the surface, i.e. taking the tangential wall velocity proportional to the wall shear, where x measures distance along the surface. In essence they derived the similarity system

$$f''' + f f'' + \beta(1 - f'^2) = 0, \tag{1}$$

subject to

$$f(0) = S, \quad f'(0) = \lambda + A f''(0), \quad f' \to 1 \quad \text{as} \quad y \to \infty, \tag{2}$$

where $\beta = 2m/(m+1)$ and primes denote differentiation with respect to y. In boundary conditions indicated in Eq. (2) S is a dimensionless transpiration velocity, with $S > 0$ for suction and $S < 0$ for blowing, A is a dimensionless velocity slip parameter, with $A \geq 0$, and λ represents the velocity of the wall relative to that of the outer flow.

Perhaps the most significant feature of the results presented in [1] was existence of a critical value λ_c of the wall velocity parameter λ, limiting solutions to $\lambda \geq \lambda_c$. The values of λ_c were seen to depend on the other dimensionless parameters, namely A and S, and had $\lambda_c < 0$. Our aim here is to extend the results given in [1], in particular by obtaining asymptotic forms for the solution to Eqs. (1) and (2) based on one of the parameters being large. Finally we note that Fauzi et al. [1] also considered the heat transfer in this situation through forced convection though here we limit attention just to the equation for the flow.

We start by considering the simplest form of the problem given by Eqs. (1) and (2), namely an impermeable fixed wall.

2. Impermeable, fixed wall, $\lambda = S = 0$

Here we solve Eq. (1) subject to

$$f(0) = 0, \quad f'(0) = A f''(0), \quad f' \to 1 \quad \text{as} \quad y \to \infty. \tag{3}$$

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In Fig. 1 we plot $f'(0)$ and $f''(0)$ against A for $\beta = 1$ (stagnation-point flow). We see that the solution starts at $A = 0$ with $f'(0) = 0$ and $f''(0) = 1.232588$ as given by the classical stagnation-point flow problem [2]. As A increases $f'(0)$ approaches an asymptotic value of 1, shown by a broken line, and that $f''(0)$ tends to zero.

With $A = 0$ we obtain the classical Falkner–Skan problem which has been shown [3] to have a critical value $\beta_{c,0}$, where $\beta_{c,0} \simeq -0.19884$, with solutions possible only for $\beta \geq \beta_{c,0}$, as can be seen in Fig. 2 for $A = 0$. The saddle-node bifurcation at $\beta = \beta_{c,0}$ gives rise to two solution branches, the upper branch continues to large positive β while the lower branch terminates in a singularity as $\beta \to 0^-$ [4]. We note, in passing, that $\beta_{c,0}$ can be determined by solving Eq. (1) subject to Eq. (2), with $A = S = 0$, and the extra condition that $f''(0) = 0$. In Fig. 2 we plot $f''(0)$ against β for representative values of A. We again see the existence of a critical value $\beta_c = \beta_c(A)$, with $|\beta_c|$ increasing as A is increased. All the curves pass through the point $\beta = \beta_{c,0}$ with $f'(0) = f''(0) = 0$. As before, the upper branch continues to large β and the lower branch terminates as $\beta \to 0$ with $f''(0) \to 0$ with the solution becoming singular in the manner described in [4].

We investigate these critical values in more detail in Fig. 3 where we plot β_c against A. We see that the curve starts with $\beta_{c,0}$ at $A = 0$ and decreases as A is increased. Our numerical integrations suggest that β_c approaches a finite limit of approximately -0.5 as A becomes large.

2.1. Solution for A large

As $A \to \infty, f \to y$ which leads us to look for a solution for A large by expanding

$$f = y + A^{-1} f_1(y) + \cdots, \tag{4}$$

where f_1 satisfies

$$f_1''' + y f_1'' - 2\beta f_1' = 0, \quad f_1(0) = 0, \quad f_1''(0) = 1, \quad f_1' \to 0 \text{ as } y \to \infty. \tag{5}$$

For a general value of β Eq. (5) can be solved in terms of confluent hypergeometric functions [5] as

$$f_1' = -\frac{\left(\beta - \frac{1}{2}\right)!}{\sqrt{2\pi}} e^{-y^2/2}\, U\left(\frac{1}{2} + \beta; \frac{1}{2}; \frac{y^2}{2}\right), \quad \text{giving } f_1'(0) = -\frac{\left(\beta - \frac{1}{2}\right)!}{\sqrt{2}\,\beta!}, \tag{6}$$

on using the notation in [5]. Expression (6) holds provided that $\beta > -\frac{1}{2}$, indicating the limiting value for β_c as $A \to \infty$ seen in Fig. 3. We next consider the effect of a moving wall.

3. Impermeable, moving wall, $\lambda \neq 0, S = 0$

For this case we solve Eq. (1) now subject to

$$f(0) = 0, \qquad f'(0) = \lambda + A f''(0), \quad f' \to 1 \text{ as } y \to \infty. \tag{7}$$

In this case we find a behaviour similar to that seen in Fig. 2 in [1] where, in effect, $f''(0)$ is plotted against λ, there for $m = 1$ and $A = 0.5$. A critical value λ_c is observed in these solutions with solutions only for $\lambda \geq \lambda_c$. The situation when $A = 0$ has been treated previously, especially for the case when $m = 0$, see for example [6, 7]. Here a critical value is again seen, $\lambda_c \sim -0.3541$ for $m = 0$ and $\lambda_c = -1.2466$ for $\beta = 1$. We investigate this case by plotting λ_c against A shown in Fig. 4a. This figure shows that λ_c is negative for all A and decreases, apparently linearly, as A is increased becoming unbounded as $A \to \infty$.

3.1. Solution for A large

The results shown in Fig. 4a suggest that we write $\lambda = \mu A$, where μ is of $O(1)$. The boundary condition on $y = 0$ then becomes $f''(0) + \mu = A^{-1} f'(0)$. So that, at leading order, we have to solve Eq. (1) now subject to $f''(0) = -\mu$. This problem leads to a critical value μ_c of μ, dependent on β. For $\beta = 0$, $\mu_c = -0.46960$ and for $\beta = 1$, $\mu_c = -1.50724$. To complete the picture in Fig. 4b we plot the critical values λ_c, obtained by solving Eq. (1) subject to $f'(0) = \lambda$, and μ_c, obtained by solving Eq. (1) subject to $f''(0) = -\mu$, both with $f(0) = 0$. The values of μ_c are shown in Fig. 4b by a broken line. In the former case we see that the λ_c curve terminates at $\beta = \beta_0 \simeq -1.325$ with $\lambda_c \to 0$, passes through the points $\beta = -1, \lambda_c = 1$ and $\beta = \beta_{c,0}, \lambda_c = 0$, with $\lambda_c > 0$ in $\beta_0 < \beta < \beta_{c,0}$. There also appears to be a gap in the curve between $\beta \simeq 0.139$ and $\beta \simeq 0.5$ (we tried to fill this gap by using a very small increment in β in the numerical integration but this did not give any results in this gap) and the curve continues to large positive β. In the latter case $\mu_c \to -\infty$ as $\beta \to -1$, has $\mu_c = 0$ at $\beta = -0.5$, μ_c is negative throughout continuing, without any gaps, to large β.

4. Permeable, fixed wall, $\lambda = 0, S \neq 0$

Here we solve Eq. (1) subject to the boundary conditions

$$f(0) = S, \qquad f'(0) = A f''(0), \quad f' \to 1 \text{ as } y \to \infty. \tag{8}$$

In Fig. 5 we plot $f''(0)$ against S for $\beta = 1$ and a range of values of A. We see that, for suction $S > 0$, the behaviour is different for $A = 0$, where $f''(0)$ grows linearly for S large, whereas for $A > 0, f''(0)$ appears to be approaching a finite value as S increases. For blowing $S < 0, f''(0)$ appears to be decreasing to zero in each case.

4.1. Solution for S large, S > 0

The plots shown in Fig. 5 indicate that we need to consider the cases $A > 0$ and $A = 0$ separately, in fact we can extend this latter case to having A of $O(S^{-1})$, where we put $A = a_0 S^{-1}$ with a_0 of $O(1)$.

4.1.1. $A = a_0 S^{-1}$

For this case we put

$$f = S(1 + S^{-2} F), \qquad \zeta = S\, y. \tag{9}$$

Applying transformation (9) in Eqs. (1) and (2) leads to

$$F''' + F'' + S^{-2}(F F'' + \beta - \beta F'^2) = 0, \qquad F(0) = 0, \quad F'(0) = a_0 F''(0), \quad F' \to 1 \quad \text{as} \quad \zeta \to \infty, \tag{10}$$

primes now denoting differentiation with respect to ζ. Eq. (10) suggests looking for an expansion for S large in the form

$$F(\zeta; S) = F_0(\zeta) + S^{-2} F_1(\zeta) + \cdots. \tag{11}$$

At leading order we find

$$F_0 = \zeta - \frac{1}{1 + a_0}(1 - e^{-\zeta}). \tag{12}$$