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Inverse simulation and experimental verification of temperature-dependent thermophysical properties☆

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ABSTRACT

An inverse method is developed to simultaneously estimate unknown temperature-dependent thermal conductivity and specific heat of a brass rod with knowledge of temperatures taken on the specimen. An experimental process with the brass rod is built for measuring temperatures at some locations to verify the thermal conductivity and specific heat. With known temperature data recorded from the experiment, inverse solutions were rapidly obtained through the Broyden–Fletcher–Goldfarb–Shanno (BFGS) combined simple step method. Results show that the proposed method can estimate thermal properties with low iterations and in a significantly short time compared to other methods. In addition, the estimated temperatures are in very good agreement with the measurement temperature. From experimental verification, the estimated thermal properties are quite close to values obtained by simple tests, with several cases of different heat generation magnitudes. The effect of measurement errors and locations, as well as measured point numbers, on the accuracy of inverse solutions was discussed. According to analysis results, the proposed method, an accurate and efficient method for the prediction of unknown thermal conductivity and specific heat, can be applied to accurately estimate thermophysical properties of various materials.

1. Introduction

Each material has a characteristic rate at which heat will flow through it. The faster the heat flows in a material, the more conductive it is. Therefore, understanding the intrinsic properties will help us choose the right materials to manage heat flow. Thermophysical properties, i.e. thermal conductivity and specific heat, not only decide thermal heat transport in the materials but also significantly affect the analysis of temperature distribution and heat flow rate when the material is heated. Thus, thermal property measurements have become important in the engineering and development of new materials. However, direct measurement of surface conditions are not feasible; thus, many researchers have devoted their studies to the inverse problem for prediction of thermal properties.

The determination of thermal properties, which are functions of temperature, from measured temperatures in the material is one kind of nonlinear inverse heat conduction problem [1]. To date various methods and experiments have been developed to determine thermal conductivity as well as heat capacity per unit volume. Several experiments for the measurement of temperature and thermal conductivity, including the hot wire method [2], laser flash method [3], hot ball method [4], and hot disk method [5], have been investigated. Recently, an experimental and numerical approach was taken to derive the thermal conductivity in a spiral coil type ground heat exchanger [6] and the thermal conductivity of bulk and dense oxynitride $Lu_4Si_2O_7N_2$ at various temperatures [7]. Many authors have studied the inverse estimation of thermal conductivity through the measurement of the temperature profile [8–13]. An unknown thermal conductivity was assumed, and the heat conduction problem was solved through iterations [8,11,13]. The first order accuracy of the finite difference method [14] and the second order finite difference technique [12] were employed to determine thermal conductivity. A simple transient method [15] and a simple measurement [16] were applied to estimate thermal conductivity and specific heat. Afterwards, these properties were simultaneously predicted by using the least-square method [17–19], semi-discretization method [9], and the Taylor series approach [14]. J. Myllymaky and D. Baroudi [20] used the finite element technique combined with the regularized output least square method, and Yang [21] used the iterative method for determining temperature-dependent thermal conductivity of a material from boundary temperature measurements. From the temperature measurements inside the material, the temperature-dependent thermal conductivity and heat capacity were also estimated by using a hybrid numerical algorithm of the Laplace transform technique, the control-volume method [22] and the conjugate gradient method [23,

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Nomenclature

A_c	surface area (m^2)
a	temperature-dependent factor of $k(T)$
Bi	Biot number
b	temperature-dependent factor of $C_p(T)$
$C_p(T)$	specific heat (J/kg°C)
D	diameter of test sample (m)
$\mathbf{E}$	updated matrix
h	convection heat transfer coefficient (W/m^2°C)
$\mathbf{H}$	Hessian matrix
$\mathbf{I}$	unit matrix
J	object function
∇J	gradient of object function
$k(T)$	thermal conductivity (W/m °C)
L	length of test sample (m)
M	number of temperature measurement points
N	number of unknown
N_i	spatial grid number
N_t	temporal grid number
$\overline{Nu}_D$	average Nusselt number
$\vec{\mathbf{P}}$	search direction
P	perimeter of test sample (m)s
Pr	Prandtl number
q	heat generation (W)
Ra_D	Rayleigh number
s	Stefan–Boltzmann constant, 5.670E − 8(W/m^{2}K^4)
t	time (s)
t_f	final time (s)
$T(z, t)$	temperature (°C)
r,z	coordinates
$\vec{\mathbf{w}}$	unknown vector

Greek symbols

σ	standard deviation of measurement error (°C)
β	search step size
λ	Lagrange function
ρ	density (kg/m^3)

Superscripts

k	value of last iteration
n	value at time points

Subscripts

inv	inverse solution
m	measurement point
∞	properties for ambient temperature

24]. D. Gossard et al. [25] studied an inverse method using a particle swarm optimization algorithm in order to find the geometrical and thermophysical properties of a three-dimensional conjugate heat transfer model. They pointed out that the thermal conductivity and the volumetric specific heat of the solid material are the two parameters that most impact thermal resistance. A hybrid method, which is a combination of the modified genetic algorithm and the Levenberg–Marquardt method, was developed by Fung-Bao Liu [26] to simultaneously designate fluid thermal conductivity and heat capacity for a transient inverse heat transfer problem. The modified, one-dimensional correction method, along with the finite volume method, was applied successfully to solve the IHCP for determining the heat transfer coefficient of one surface of a flat plate based on the thermographic temperature measurements of the opposite surface [27]. In addition, the thermal conductivity in a one-dimensional heat conduction problem was successfully estimated by applying the finite difference method and linear least-squares-error method [28].

Although many optimization methods have been applied to the inverse problem for the solution of thermal conductivity, as well as heat capacity or specific heat, all researches were simulated to get the inverse results expected from the work in reference [16]. The authors estimated the constant thermal diffusivity and thermal conductivity by using the inverse method with an experiment to measure temperature. However, during the experiment the heat convection, which always occurs, was not considered. In this study, an experimental transient temperature that took into account the convection and radiation of the surrounding environment was established to estimate temperature-dependent thermal conductivity and specific heat through solving the inverse heat transfer problem. Moreover, the BFGS method [29], that very few authors have used before [30] to predict thermal properties, combined the simple step method for estimation of unknown quantities.

2. Method and mode definition

2.1. Mathematical model

In order to illustrate the method for use in simultaneously determining unknown temperature-dependent thermal conductivity, $k(T)$, and heat capacity per unit volume, $C_p(T)$, in a material, a transient inverse heat transfer problem is considered. A brass rod, with diameter D and length L, for simulation as well as experimentation is supplied with a fixed heat source at the end allowing natural convection as well as radiation, as shown in Fig. 1. The natural convection depends on the variation in surface temperature and thermophysical properties, and heat transfer relations in natural convection are based on experiments. The geometry of the problem in this study is a cylinder with horizontal convection. Therefore, the heat transfer coefficient is determined in accordance with the average Nusselt number [31] as follows:

$$\overline{Nu}_D = \left\{0.60 + \frac{0.387 Ra_D^{1/6}}{\left[1+(0.559/Pr)^{9/16}\right]^{8/27}}\right\}^2 \quad \text{for } Ra_D \le 10^{12} \tag{1}$$

where Pr and Ra_D are the Prandtl number and the Rayleigh number, respectively. The calculation of the heat transfer coefficient is defined in the following equation:

$$h = \frac{k\overline{Nu}_D}{D} \tag{2}$$

where h is the heat transfer coefficient. In this study, the cylindrical rod which is chosen for simulating and testing, is made from brass with a high thermal conductivity. The Biot number is [31] $Bi = hD/2k \ll 0.1$, so the variation in temperature in the direction of the radius (along the r axis) is very slight compared to the z direction. Therefore, according to the conservation of heat energy and Fourier's law, the partial differential heat transfer equation and associated initial and boundary conditions can be written as:

$$\begin{aligned}\rho C_p(T)\frac{\partial T}{\partial t} = {} & k(T)\frac{\partial^2 T}{\partial z^2} \\ & + \frac{\partial k(T)}{\partial T}\left(\frac{\partial T}{\partial z}\right)^2 - \frac{P}{A_c}\left(h(T)(T-T_\infty) + s\left(T^4 - T_\infty^4\right)\right) \\ & + qH(z-z_q)\end{aligned} \tag{3}$$

with initial condition:

$$T(t,z) = T_\infty \quad \text{at} \quad t = 0 \tag{4}$$

and boundary conditions:

$$k(T)\frac{\partial T(t,0)}{\partial z} = h(T)(T(t,0)-T_\infty) + s\left(T^4(t,0) - T_\infty^4\right) \text{at } z = 0 \tag{5}$$