International Communications in Heat and Mass Transfer 71 (2016) 157–163

Contents lists available at ScienceDirect

International Communications in Heat and Mass Transfer

journal homepage: www.elsevier.com/locate/ichmt

Constructal optimization for leaf-like body based on maximization of heat transfer rate☆

Lingen Chen [a,b,c,*], Huijun Feng [a,b,c], Zhihui Xie [a,b,c], Fengrui Sun [a,b,c]

[a] Institute of Thermal Science and Power Engineering, Naval University of Engineering, Wuhan 430033, PR China
[b] Military Key Laboratory for Naval Ship Power Engineering, Naval University of Engineering, Wuhan 430033, PR China
[c] College of Power Engineering, Naval University of Engineering, Wuhan 430033, PR China

ARTICLE INFO

Available online 30 December 2015

Keywords:
Constructal theory
Leaf-like body
Maximum heat transfer rate
Generalized thermodynamic optimization

ABSTRACT

A model of a first order leaf-like body (FOLLB), which is assembled by many elemental leaf-like bodies (ELLBs), is established in this paper by using Constructal theory. The maximum dimensionless heat transfer rate (DHTR) and the optimal structure of the FOLLB are obtained. It shows that there exist an optimal dimensionless cross-sectional area of the elemental vein, an optimal dimensionless thickness of the FOLLB, as well as an optimal number of ELLBs in the FOLLB which lead to the triple maximum DHTR of the FOLLB. The heat transfer performance (HTP) of the FOLLB becomes better with the increases in Biot number and the thermal conductivity ratio. When the optimal construct of the FOLLB was compared with that of the ELLB, it was found that the maximum DHTR of the FOLLB is increased by 69.83% for the same body volume, and the HTP of the body can be greatly improved.

1. Introduction

Fin is one of the typical tools to enhance heat transfer. The Constructal theory [1–13] is a powerful tool in solving various optimization problems, such as heat conduction problems [14–21], heat convection problems [22–30], mass transfer problems [31–33] as well as combined heat and mass transfer problems [34–36]. Bejan and Dan [37] firstly applied this theory into the performance optimization of the tree-shaped fin, and obtained the maximum heat transfer rate (MHTR) between a point and a volume of the fin. Henceforth, the investigations on the heat transfer optimizations for various fins are the hotspots in the heat transfer field.

A great variety of typical fins had been investigated by many scholars, such as T-shaped fins [38–43], Y-shaped fins [44–50], rectangular fins [51–53], T–Y simple and complex assembly of fins [54,55], umbrella-shaped fins [39], pin fins [56–58], annular fins [59–61], leaf-like fin [62], as well as plate-fins [63,64]. Moreover, the fin models gradually tended to be practical and complex. Kundu and Bhanja [42] and Das [53] investigated the fins with variable physical properties, and these fins were the more practical ones compared with those with constant physical properties. Sharqawy and Zubair [57,59] investigated the spine and annular fins under wet condition, and concluded that the fin efficiency increased when the pressure of the atmosphere increased. Combelles et al. [62] built a model of an elemental leaf-like body (ELLB) assembled by a vein and a blade with different thermal conductivities. The MHTR at the root of the vein was obtained by searching for optimal design of the ELLB. Kim et al. [63,64] carried out performance analysis of a variable thickness plate-fin based on volume averaging theory. The results showed that the fin's performance could be improved by increasing its thickness, and this reduction could reach to 15% in the water-cooled heat sink case. Bhanja and Kundu [65] carried out performance comparisons of the T-shaped porous fin and solid fin. They concluded that the heat transfer performance (HTP) of the porous fin was superior to that of the solid one, and this conclusion was also illustrated in Refs. [66–69].

Based on the model of the ELLB in Ref. [62], a model of a first order leaf-like body (FOLLB) will be built in this paper. The FOLLB is composed of a first order vein (FOV) and many ELLBs. Constructal optimization of the FOLLB will be implemented, and the dimensionless heat transfer rate (DHTR) will be maximized. HTP comparisons of the ELLB in Ref. [62] and the FOLLB in this paper will be carried out.

2. First order leaf-like body model

A model of the ELLB as shown in Fig. 1 was built in Ref. [62]. It is composed of an elemental blade ($2H_0 \times L_0 \times t$) and an elemental vein ($A_0 \times L_0$). The heat conduction differential equation and the corresponding boundary conditions of the elemental vein (EV) are, respectively, given as [62]

$$\frac{d}{dx}\left(k_p A_0 \frac{dT_0}{dx}\right) = 2.516 k_0 (T_0 - T_{\min}) \left(\frac{ht}{k_0}\right)^{1/2}, \tag{1}$$

☆ Communicated by W.J. Minkowycz

* Corresponding author at: Institute of Thermal Science and Power Engineering, Naval University of Engineering, Wuhan 430033, PR China.
E-mail addresses: lgchenna@yahoo.com, lingenchen@hotmail.com (L. Chen).

http://dx.doi.org/10.1016/j.icheatmasstransfer.2015.12.022
0735-1933/

Nomenclature

A_0	cross-sectional area of the elemental vein, m^2
A_1	cross-sectional area of the first order vein, m^2
Bi	Biot number
H_0	width of the ELLB, m
H_1	width of the FOLLB, m
h	heat transfer coefficient of the blade, $W/m^2/K$
k_0	thermal conductivity of the blade, $W/m/K$
k_p	thermal conductivity of the vein, $W/m/K$
$\tilde{k}$	ratio of the thermal conductivities, k_p/k_0
L_0	length of the ELLB, m
L_1	length of the FOLLB, m
$M_{1,i}$ *ith*	node along the length direction of the first order vein
n_1	number of the ELLBs
$q_{0,i}$	heat current at the root of the elemental vein, W
$q_{1,i}$	heat current before the node of $M_{1,i}$, W
q_1	heat current at the root of the FOLLB, W
$T_{0,m}$	temperature at the root of the first order vein, K
$T_{1,i}$	temperature at the root of the elemental vein, K
T_∞	environment temperature, K
t	thickness of the FOLLB, m
V_1	volume of the FOLLB, m^3

Subscripts

m/mm/mmm	single/double/triple minimized
opt/oo/ooo	optimal/twice/three times optimized

$$\frac{dT_0}{dx} = 0, x = 0, \tag{2}$$

$$T = T_{0,m}, x = L_0, \tag{3}$$

where k_0 and k_p are, respectively, the thermal conductivities of the blade and vein, $T_{0,m}$ is the temperature at the root of the vein, h is the heat transfer coefficient, and $a_1 = 2.516\frac{(k_0 ht)^{1/2}}{k_p A_0}$.

Solving Eqs. (1)–(3), the maximum heat current (q_0) entering the ELLB subjected to the total body volume constraint can be derived [62]:

$$q_0 = k_p A_0 \left(\frac{dT_0}{dx}\right)_{x=L_0} = k_p A_0 (T_{0,m} - T_\infty) \tanh(a_1 L_0). \tag{4}$$

The geometry of the ELLB can be varied, and the corresponding optimal shape of the ELLB is [62]:

$$\frac{t}{H_0} = 0.996\left(\frac{ht}{k_0}\right)^{1/2}. \tag{5}$$

A FOLLB, which is assembled by an even number (n_1) of ELLBs in Fig. 1, is shown in Fig. 2. These ELLBs are distributed on the both sides of a FOV ($A_1 \times L_1$, k_p) and are connected by this vein. The FOV extends from its root to the roots of the two farthest EVs, therefore, the length of the FOV is $L_1 = (n_1/2) \cdot (2H_0) - H_0$. The nodes located at the roots of the EVs along the length direction of FOV are $M_{1,i}$ ($i = 1, 2, \ldots, n_1/2$, and $n_1/2$ is the number of the nodes), respectively. The heat current q_1 enters the root of the FOV, and flows along the FOV firstly (the heat current after the node $M_{1,i-1}$ and before the node $M_{1,i}$ along the length direction of the FOV is $q_{1,i}$, $i = 1, 2, \ldots, n_1/2$). Then, parts of the heat current (the heat current is $q_{0,i}$, $i = 1, 2, \ldots, n_1/2$) enters the ELLBs from the root of the EV, and the corresponding temperature at this root (i.e., at the node $M_{1,i}$) is $T_{1,i}$ ($i = 1, 2, \ldots, n_1/2$). The heat transfer mechanism in the ELLBs is shown in Fig. 1. The side surface of the first order blade and the top of the FOV are adiabatic. For the purpose of simplification, one can assume that the heat exchange between the FOV and the ELLB only occurs at the root of the EV, and the adjacent ELLBs do not exchange heats with each other. Because the cross-sectional area (CSA) for longitudinal conduction along the FOV is much small than the surface area of the first order blade, the heat of the FOV dispersed directly into the ambient is ignored. In this case, the geometric parameters (the number (n_1) of ELLBs, the CSAs (A_0 and A_1) of the EV and FOV, the width H_0, length L_0 and thickness t) of the FOLLB can be varied.

From Eq. (4), the tributary heat current ($q_{0,i}$) entering the root of each EV is:

$$q_{0,i} = k_p A_0 a_1^{1/2} (T_{1,i} - T_\infty) \tanh\left(a_1^{1/2} L_0\right) (1 \le i \le n_1/2 - 1). \tag{6}$$

Consider the contiguous nodes ($M_{1,i}, M_{1,i+1}$) along the FOV, the heat currents between $M_{1,i}$ and $M_{1,i+1}$ can be given by:

$$q_{1,i} = 2q_{0,i} + q_{1,i+1}\ (1 \le i \le n_1/2 - 1), \tag{7}$$

$$q_{1,n_1/2} = 2q_{0,n_1/2}, \tag{8}$$

$$q_{1,i} = k_p A_1 \frac{(T_{1,i-1} - T_{1,i})}{2H_0} (2 \le i \le n_1/2), \tag{9}$$

$$q_1 = k_p A_1 \frac{(T_{1,m} - T_{1,1})}{H_0}, \tag{10}$$

where $q_{1,1} = q_1$.

The total volume of the FOLLB can be given by:

$$V_1 = n_1 (A_0 + 2H_0 t) L_0 + A_1 H_0 (n_1 - 1). \tag{11}$$

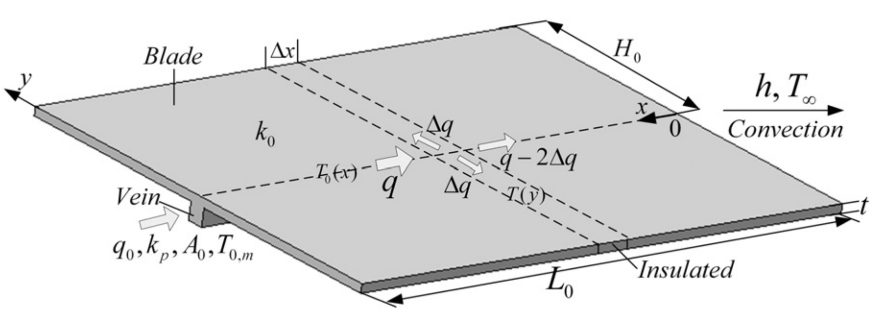

Fig. 1. Elemental leaf-like body [62].