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# Nonlinear Study of Kelvin-Helmholtz instability of cylindrical flow with mass and heat transfer☆

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ABSTRACT

In this work, a viscous potential flow theory is used to study the nonlinear Kelvin-Helmholtz instability of the interface between two viscous, incompressible and thermally conducting fluids, when the phases are enclosed between two horizontal cylindrical surfaces coaxial with the interface and when there is mass and heat transfer across the interface. The method of multiple time expansions is applied and it is shown that the evolution of amplitudes is governed by a nonlinear first order partial differential equation. The various stability criteria are discussed both analytically and numerically, stability diagrams are studied graphically. It is observed that the heat and mass transfer has destabilizing effect on the stability of the system in the nonlinear analysis.


## 1. Introduction

In recent years, a great deal of interest is focused on the understanding of stability of fluids flow in a cylindrical geometry because cylindrical geometry is associated with the problems related to liquid jets, cooling of rotating machinery, petroleum industry and cooling of fuel rods by liquid coolants in the nuclear reactor etc. Wu and Wang [1] studied the Kelvin-Helmholtz instability in a cylindrical flow with a shear layer and found that a shear layer suppresses pinching modes more effectively. Joseph et al. [2] considered the Kelvin-Helmholtz instability in a cylindrical geometry of viscous fluid into the same fluid. Awasthi and Agrawal [3] studied the Kelvin-Helmholtz instability in a cylindrical flow when both fluids are of different viscosity. The instability of a liquid jet into a gas or another liquid was considered by Funada et al. [4]. They found that the instability is driven by Kelvin–Helmholtz instability due to a velocity difference and a Neck down due to capillary instability.

The heat and mass transfer phenomenon in multiphase flows has received much attention in recent years because of its wide range of applications in many situations such as boiling heat transfer in chemical engineering and in geophysical problems. Linear stability analysis of the physical system consisting of a vapor layer underlying a liquid layer of an inviscid fluid was carried out by Hsieh [5]. He observed that the heat and mass transfer phenomenon enhances the stability of the system if the vapor layer is hotter than the liquid layer. Nonlinear Kelvin-Helmholtz instability of liquid-vapor interface in a plane geometry of an inviscid fluid was performed by Lee [6]. He concluded that when the fluids are inviscid, the linear stability analysis does not affected by heat transfer coefficient but it plays a crucial role in the nonlinear stability analysis.

The effect of heat and mass transfer on the Kelvin-Helmholtz instability of miscible and viscous fluids using viscous potential flow theory was considered by Asthana and Agrawal [7]. They observed that heat and mass transfer has a strong stabilizing effect when the lower fluid is highly viscous and a weak destabilizing effect when the fluid's viscosity is low. Awasthi and Agrawal [8] studied the effect of heat and mass transfer on the Rayleigh-Taylor instability in a plane geometry. Kim et al. [9] investigated the capillary instability including the effect of interfacial heat and mass transfer and noted that the interfacial heat and mass transfer phenomenon resists the growth of disturbance waves. Asthana et al. [10] considered the effect of heat and mass transfer on Kelvin-Helmholtz instability of viscous and miscible fluids in a cylindrical geometry.

The second and higher order terms of perturbed quantities are neglected in the linear theory and therefore, a uniform model based on the linear theory is insufficient to explain the mechanism involved, and hence, the nonlinear theory is needed to reveal the effect of heat and mass transfer on the stability of the system. The nonlinear Rayleigh-Taylor instability of inviscid fluids in plane geometry was considered by Hsieh [11]. He included the effect of heat and mass transfer into the analysis and concluded that nonlinearity increases the stability range when there is heat and mass transfer across the interface. The nonlinear analysis of Rayleigh-Taylor instability in a cylindrical geometry with mass and heat transfer was studied by Lee [12]. Lee [13] investigated the effect of heat and mass transfer on the Kelvin-Helmholtz instability of inviscid fluids in a cylindrical geometry. He observed that in the linear inviscid analysis heat and mass transfer has no effect on stability criterion while it plays an important role in the nonlinear analysis.

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**Nomenclature**

| | |
|---|---|
| $\rho$ | Density ($gm/cm^3$) |
| $\mu$ | Viscosity ($poise$) |
| $U$ | Velocity ($cm/s$) |
| $h$ | Thickness ($cm$) |
| $R$ | Radius ($cm$) |
| $\sigma$ | Surface Tension ($dyne/cm$) |
| $T$ | Temprature (°C) |
| $\eta$ | Surface Elevation ($cm$) |
| $F$ | Free Surface ($cm$) |
| $L$ | Latent Heat ($J/g$) |
| $K$ | ThermalConductivity ($Wm^{-1}.K^{-1}$) |
| $\phi$ | Velocity Potential |

The irrotational theory which includes the effect of normal viscous stresses at the interface of two viscous fluids is called viscous potential flow (VPF) theory. The viscous potential flow theory plays a crucial role while studying the nonlinear interfacial stability. Elcoot [14] applied viscous irrotational theory to study the nonlinear capillary instability when fluids are subjected to an axial electric field. Sirwah [15] studied the nonlinear Kelvin-Helmholtz instability in plane geometry in the presence of tangential magnetic field. Awasthi and Agrawal [16] studied the nonlinear effects on the capillary instability when the fluids are miscible and viscous and concluded that the nonlinearity reduces the region of stability. Awasthi [17] investigated the nonlinear Rayleigh-Taylor instability of plane interface between two viscous and miscible fluids including the effect of heat and mass transfer. Awasthi [18] studied the nonlinear Rayleigh-Taylor instability of cylindrical flow using viscous potential flow theory.

In the present work, an attempt has been made to study the nonlinear Kelvin-Helmholtz instability of the interface between two incompressible and viscous fluids, when the phases are enclosed between two horizontal cylindrical surfaces coaxial with the interface and when there is heat and mass transfer across the interface. The multiple time expansion method has been used for the investigation and the evolution of amplitude is shown to be governed by a nonlinear first order partial differential equation. Stability is discussed theoretically as well as numerically and stability region is displayed graphically. In addition, a comparative analysis has been made between the results obtained in the inviscid flow analysis (Lee [13]) and present viscous flow analysis.

## 2. Problem Formulation

Consider the parallel flow of two incompressible, thermally conducting and viscous fluids, separated by a cylindrical interface $r=R$ in an annular configuration as presented in Fig. 1. We consider a cylindrical system of coordinates $(r,\theta,z)$, so that in the equilibrium state $z-$axis is the axis of symmetry of the system. The inside fluid (1) occupies the inner region $R_1<r<R$, having thickness $h_1$, density $\rho^{(1)}$, velocity $U_1$, viscosity $\mu^{(1)}$ and is bounded by the rigid cylindrical surface $r=R_1$ while the outside fluid (2) occupies the outer region $R<r<R_2$, having thickness $h_2$, density $\rho^{(2)}$, velocity $U_2$, viscosity $\mu^{(2)}$ and is bounded by the rigid cylindrical surface $r=R_2$, where $h_1=r-R_1$ and $h_2=R_2-r$. The temperatures at $r=R_1, r=R$and $r=R_2$are $T_1, T_0$ and $T_2$, respectively and surface tension at the interface is taken as $\sigma$. We assume that both fluids are incompressible and irrotational. In the basic state, thermodynamics equilibrium is assumed and the interface temperature $T_0$ is set equal to the saturation temperature.

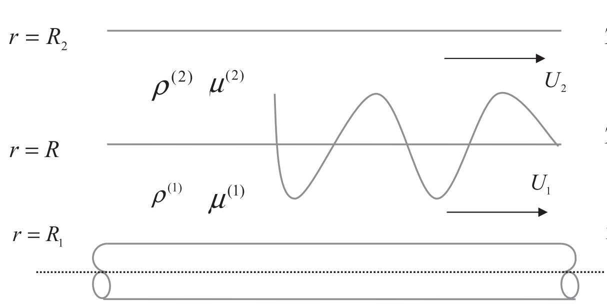


**Fig. 1.** The equilibrium configuration of the fluid system.

On applying the small disturbances to the equilibrium state, the interface can be expressed as

$$F(r,z,t) = r-R-\eta(z,t) = 0 \tag{2.1}$$

where $\eta$ denotes the varicose interface displacement, and for which the outward unit normal vector is given by

$$\boldsymbol{n}=\frac{\nabla F}{|\nabla F|}=\left\{1+\left(\frac{\partial\eta}{\partial z}\right)^2\right\}^{-1/2}\left(\boldsymbol{e_r}-\frac{\partial\eta}{\partial z}\boldsymbol{e_z}\right) \tag{2.2}$$

where $\boldsymbol{e_r}$ and $\boldsymbol{e_z}$ are unit vectors along the $r$ and $z$ directions, respectively.

Our analysis is based on the potential flow theory, therefore; velocity can be expressed as the gradient of the potential function i.e.

$$\boldsymbol{u_j} = \nabla\phi^{(j)}, \quad (j=1,2) \tag{2.3}$$

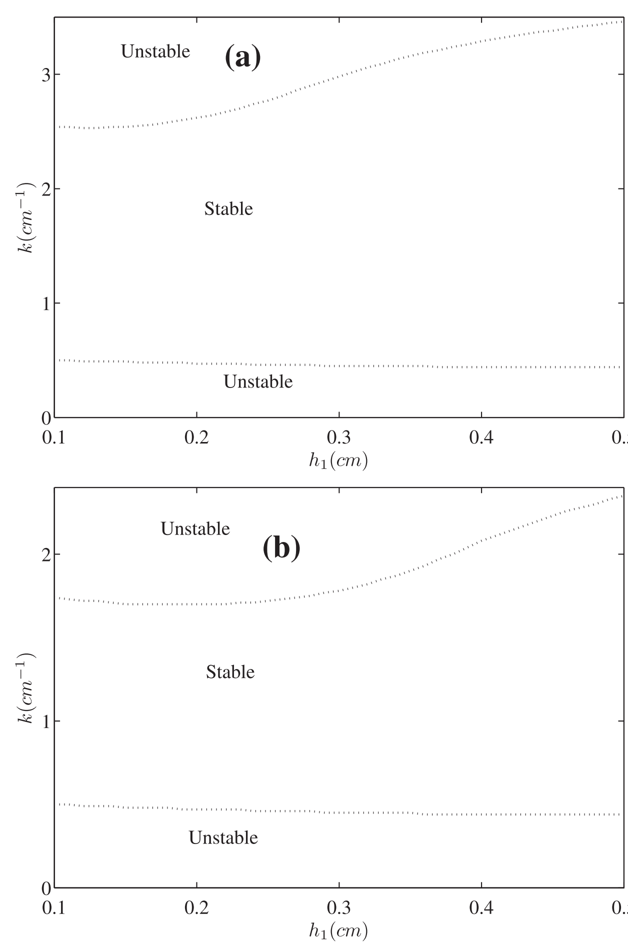


**Fig. 2.** The stability diagram for water-vapor system when $\rho^{(1)}=0.001gm/cm^3$,$\rho^{(2)}=1.0gm/cm^3$,$R_1=1cm$,$R_2=2cm$, (a) $\alpha=1.0gm/cm^3s$ (b) $\alpha=2.0gm/cm^3s$.