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Investigation into the influences of the low speed's accuracy on the hypersonic heating computations☆

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ABSTRACT

With the rapid development of the Computational Fluid Dynamics (CFD), it is in a high demand and still remains unresolved to compute hypersonic heating accurately. In the past years, many researchers tried to solve this problem by studying the upwind scheme's choice. However, most of them just focused on the following two aspects: the scheme's level of robustness against the shock anomaly and the scheme's resolution in capturing discontinuity. Few people relate this problem to the scheme's level of accuracy at low speeds. In this paper, we conduct a systematic study on this issue. Results in our test cases show that a high level of accuracy at low speeds is beneficial to the hypersonic heating computations. Also, the AUSMPWM scheme performs well in hypersonic heating computations.

1. Introduction

The past years have witnessed tremendous growth in Computational Fluid Dynamics (CFD). However, it is still hard to predict surface heating accurately in hypersonic computations [1]. In general, researchers studied this problem from many aspects, such as the computational mesh [2], the computational scheme, and the physical modeling [3], etc.

In terms of the computational scheme, many researchers attributed this problem to the upwind schemes' two major defects: low levels of resolution in capturing discontinuity and low levels of robustness against the shock anomaly [4,5]. However, many upwind schemes' levels of accuracy in hypersonic heating predictions are still unsatisfactory in the following cases: the normal spacing at the wall is small enough [6] and the shock anomaly is dispelled by generating the computational mesh seriously.

Up to now, Roe's FDS [7] and AUSM+ [8] have won high praises and been widely used in industry. Many well-known codes, such as OVERFLOW and LAURA, have options to use them. However, researches find that they are essentially of the same form and not applicable to low speeds' computations [9,10]. It is a traditional way to combine these upwind schemes with a preconditioning matrix at low speeds [11–15]. But they will return to their original forms at supersonic speeds due to the global cut-off problem [16,17]. Thus, the preconditioning methods are of no help to the hypersonic heating computations. When a hypersonic computation is conducted by using such shock capturing methods, the accuracy in the low speeds' zones may be deteriorated, especially in boundary layers which are filled with low speeds' flows and critical for the hypersonic heating computations in the authors' opinions. Unfortunately, few people notice this issue.

To be with high levels of accuracy at low speeds and free from the cut-off problem, some researchers made a thorough mathematical study and proposed some new schemes [5,16]. For example, Shima and his co-workers found that the dissipation in the AUSM type schemes' pressure flux might be too large at low speeds. To dispel this defect, they adopted a function to monitor the dissipation and proposed SLAU [18]. Ref. [15] shows that the SLAU scheme performs much better than AUSM+ at low speeds. Moreover, the authors proposed the AUSMPWM scheme which is with a higher level of accuracy at low speeds [11]. On the other hand, Fillion improved the original Roe scheme by centering the pressure gradient and proposed the F-Roe scheme [19]. Numerical tests show that the F-Roe scheme is also with a high level of accuracy at low speeds [17].

Theoretically, the F-Roe scheme is the same as the original Roe scheme at supersonic speeds. Also, the AUSMPWM scheme returns to the AUSMPW+ scheme at high speeds [20]. Thus, the AUSMPWM scheme and the F-Roe scheme can be regarded as the improvements for the AUSMPW+ scheme and the original Roe scheme at low speeds. And they should perform better in hypersonic heating computations due to their lower dissipations and higher resolutions. However, to the best of the authors' knowledge, there is still no thorough study on

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this issue, especially when they are combined with different limiters or different computational meshes. In fact, these properties are so important in practical simulations that people should pay special attention to them.

This paper is organized as follows. In the second section, we will briefly overview the Navier–Stokes equations. Also, an introduction to the upwind schemes aforementioned will be presented in this section. Section 3 will give some test cases. The last section contains concluding remarks.

2. Navier–Stokes equations and upwind schemes

2.1. Navier–Stokes equations

Navier–Stokes equations are usually used to describe the motion of the viscous flows. It can be written in the following form:

$$\frac{\partial \mathrm{q}}{\partial t}+\nabla\cdot \mathrm{F}(\mathrm{q})-\nabla\cdot \mathrm{F}_v(\mathrm{q},\nabla \mathrm{q})=\mathbf{0} \tag{1}$$

where $\boldsymbol{q}$ is the conservative variable vector, $\boldsymbol{s}$ is the auxiliary variable vector, $\boldsymbol{F}(\boldsymbol{q})$ is the inviscid flux vector and $\boldsymbol{F}_v(\boldsymbol{q},\boldsymbol{s})$ is the viscous flux vector.

For two-dimensional cases, we have

$$\mathrm{q}=(\rho,\rho u,\rho v,\rho e)^T \tag{2}$$

$$\mathrm{F}(\mathrm{q})=\begin{pmatrix}\rho u\\ \rho u^2\\ \rho uv\\ (\rho e+p)u\end{pmatrix}\vec{i}+\begin{pmatrix}\rho v\\ \rho uv\\ \rho v^2\\ (\rho e+p)v\end{pmatrix}\vec{j} \tag{3}$$

$$\mathrm{F}_v(\mathrm{q},\nabla \mathrm{q})=\begin{pmatrix}0\\ \tau_{xx}\\ \tau_{xy}\\ u\tau_{xx}+v\tau_{xy}+b_x\end{pmatrix}\vec{i}+\begin{pmatrix}0\\ \tau_{xy}\\ \tau_{yy}\\ u\tau_{xy}+v\tau_{yy}+b_y\end{pmatrix}\vec{j} \tag{4}$$

$$\tau_{xx}=2\mu\frac{\partial u}{\partial x}+\left(\beta-\frac{2}{3}\mu\right)\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right) \tag{5}$$

$$\tau_{yy}=2\mu\frac{\partial v}{\partial y}+\left(\beta-\frac{2}{3}\mu\right)\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right) \tag{6}$$

$$\tau_{xy}=\mu\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right) \tag{7}$$

$$b_x=\kappa\frac{\partial T}{\partial x} \tag{8}$$

$$b_y=\kappa\frac{\partial T}{\partial y} \tag{9}$$

where ρ is density, u and v are velocity vector components, p is static pressure, e is internal energy, T is temperature, μ is the dynamic viscosity, β is the bulk viscosity, and κ is the thermal conductivity coefficient.

2.2. Upwind schemes

2.2.1. The original Roe scheme [7]

Based on the Godunov's idea [21], Roe linearized the Jacobian matrices by three rules in his paper. It is a three-wave approximate Riemann solver and FDS type scheme with the following form.

$$\mathbf{F}_{1/2}=\frac{1}{2}(\mathbf{F}_L+\mathbf{F}_R)-\frac{1}{2}\mathbf{R}\left|\hat{\mathbf{\Lambda}}\right|\mathbf{L}\Delta \mathrm{q}. \tag{10}$$

The (^) stands for Roe-averaged values, **R** and **L** are right and left eigenvectors, respectively, and **Λ** is the diagonal matrix of characteristic speeds.

2.2.2. F-Roe [19]

Realizing the idea of centering the pressure gradient, Fillion proposed the F-Roe scheme by adding a pressure correction to the momentum flux in Eq. (10) as follows:

$$\mathbf{F}_{1/2}=\frac{1}{2}(\mathbf{F}_L+\mathbf{F}_R)-\frac{1}{2}\mathbf{R}\left|\hat{\mathbf{\Lambda}}\right|\mathbf{L}\Delta \mathrm{q}+\frac{1}{2}[1-f(M)]\begin{bmatrix}0\\ \rho a n_x\Delta U\\ \rho a n_y\Delta U\\ 0\end{bmatrix} \tag{11}$$

where

$$f(M)=\min(M,1)$$

$$U\ =\ u\cdot n_x+v\cdot n_y$$

and a is the numerical sound speed.

2.2.3. AUSMPW+ [8]

Kim proposed the AUSMPW+ scheme which split the inviscid flux into an advection term and a pressure term, respectively. It is widely used featuring simplicity and robustness against the shock-related anomaly.

The AUSMPW+ scheme can be written in the following form:

$$\begin{aligned}&\mathbf{F}_{1/2}=\overline{M}_L^+a_{1/2}\mathbf{\Phi}_L+\overline{M}_R^-a_{1/2}\mathbf{\Phi}_R+\mathbf{P}_{1/2},\mathbf{\Phi}=(\rho,\rho u,\rho v,\rho H)^T\\ &\mathbf{P}_{1/2}=\left(P_L^+\big|_{\alpha=0}\mathbf{P}_L+P_R^-\big|_{\alpha=0}\mathbf{P}_R\right),\ \mathbf{P}=(0,p,0,0)^T\end{aligned} \tag{12}$$

$$\begin{aligned}&\text{If}\ \ \mathrm{m}_{1/2}=M_L^++M_R^-\geq 0,\\ &\quad\overline{M}_L^+=M_L^++M_R^-\cdot[(1-w)\cdot(1+f_R)-f_L]\\ &\quad\overline{M}_R^-\ =\ M_R^-\cdot w\cdot(1+f_R)\end{aligned} \tag{13}$$

otherwise,

$$\begin{aligned}&\overline{M}_L^+=M_L^+\cdot w\cdot(1+f_L)\\ &\overline{M}_R^-\ =\ M_R^-+M_L^+\cdot[(1-w)\cdot(1+f_L)-f_R]\end{aligned} \tag{14}$$

where

$$\begin{aligned}&w=1-\min\left(\frac{p_L}{p_R}\right)^3,\\ &f_{L,R}=\begin{cases}\left(\dfrac{p_{L,R}}{p_S}-1\right)\min\left(1,\dfrac{\min(p_{1,L},p_{1,R},p_{2,L},p_{2,R})}{\min(p_L,p_R)}\right)^2, & P_S\neq 0\\ 0,\ otherwise\end{cases}\\ &P_S=P_L^+\mathbf{P}_L+P_R^-\mathbf{P}_R\end{aligned} \tag{15}$$

$$M_{L/R}^{\pm}(Ma)=\begin{cases}\pm\dfrac{1}{4}(Ma\pm 1)^2\pm\alpha\left(Ma^2-1\right)^2, & |Ma|<1\\ \dfrac{1}{2}(Ma\pm|Ma|), & |Ma|\geq 1\end{cases} \tag{16}$$

$$P_{L/R}^{\pm}(Ma)=\begin{cases}\dfrac{1}{2}\dfrac{Ma\pm|Ma|}{Ma} & |Ma|\geq 1\\ \dfrac{1}{4}(Ma\pm 1)^2(2\mp Ma)\pm\beta Ma\left(Ma^2-1\right)^2 & |Ma|<1\,.\end{cases} \tag{17}$$

More details can be found in the Ref. [22].

2.2.4. AUSMPWM [11]

The AUSMPW+ scheme is for the compressible solver and meets the problem of large disparity between the fluid speed and the acoustic speed, which may leads to deteriorated accuracy in low speeds'