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Method of lines in nonlinear frictional heating during braking☆

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ABSTRACT

The boundary-value heat conduction problem for two semi-infinite bodies compressed by constant pressure and sliding with a constant deceleration has been formulated. Due to friction at the contact surface the heat is generated. It is assumed that the materials of the bodies are thermal sensitive, i.e. their thermophysical properties are temperature dependent. This problem allows determining temperature fields in such elements of brakes, as a pad and a disk. At the first stage, the Kirchhoff transformation has been used for partial linearization of the nonlinear problem. Next, the Kirchhoff functions have been found by the method of lines. The transition has been made from the Kirchhoff functions to temperatures on the basis of formulas obtained for the friction couple aluminum alloy series (a disk) — the metal-ceramic (a pad). For the case of braking with a constant deceleration and at constant pressure the comparison of the results obtained with the proposed approach, and on the basis of an iterative procedure has been executed. The influence of duration of pressure increase on the temperature has been investigated.

1. Introduction

To improve accuracy of calculations of the brake systems strength to reduce the margin of safety, it is necessary to determine the temperature fields more accurately, which, in most cases, is impossible without the study of non-linear models [1]. The models involve, in particular, the boundary-value problems of heat conduction, which should take into account the dependences on the temperature of the coefficients of friction, the thermal conductivity, the specific heat and the heat transfer. The nonlinearity is necessary to consider because the range of temperatures, in which braking occurs, is expanded, and because there is the rise in temperatures of the working surfaces of a pad and a disk, caused by the increase of speeds of sliding and specific capacity of friction. In such circumstances the solution of linear heat problem of friction during braking can not only give a significant quantitative error, but it may also lead to wrong conclusions. Methods of solving the linear heat problems of friction are much better developed than nonlinear [2–4]. One of the most common techniques used to solve the nonlinear problems is to reduce the nonlinear problem to a linear (linearization) by means of the Kirchhoff substitution [5]. As a result, the initially nonlinear boundary heat conductivity problem is completely linearized or reduced to nonlinear problems of such type, for which the methods of solving are known.

In the case of heat problems of friction at braking with imperfect thermal contact of the pad and the disk, equations of heat conductivity of parabolic type linearize completely (for materials with a simple nonlinearity) or partially (for materials with arbitrary nonlinearity), as a result of the Kirchhoff substitution. In addition, the boundary conditions of imperfect heat contact, which take into account the thermal resistance of the contact, become nonlinear. The review of some methods of nonlinear heat problems of friction solution is presented in article [6], which focuses on the method of successive approximations and the method of lines. In the method of successive approximations (iterations) the solution of the corresponding linear problem has been adopted as the initial approximation, and then, by the use of specially chosen operator on each step, the solution obtained in the previous step is clarified. Some variants of iteration methods relating to the solution of the one-dimensional heat problem of friction in the case of materials with the simple nonlinearity have been tested in articles [7–9], and for materials with any nonlinearity — in article [10]. It should be noted that in these articles the frictional heating during braking with a constant deceleration were studied.

However, the most effective ways to solve the nonlinear problems are based on various numerical methods and, particularly, on the finite-difference schemes — explicit, implicit, explicit–implicit, et al. [11,12]. Very often, before obtaining the solution to the nonlinear boundary-value problem, it is transformed by substitutions and then by using one of the difference schemes. Such an algorithm of solving a

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Nomenclature

a	characteristic dimension
Bi	Biot number
c	specific heat
c_0	specific heat at an initial temperature
f	friction coefficient
h	coefficient of thermal conductivity of contact
K	coefficient of thermal conductivity
K_0	coefficient of thermal conductivity at an initial temperature
k	coefficient of thermal diffusivity
p	pressure
q	specific power of friction
T	temperature
T_0	initial temperature
T^*	dimensionless temperature
t	time
t_m	duration of the pressure increase of the loading from zero to nominal value
t_s^0	duration of braking at constant pressure
t_s	braking time
V	sliding speed
V_0	initial sliding speed
z	spatial coordinate
Θ	Kirchhoff's function
ρ	specific density
τ	dimensionless time in the case of constant pressure
τ_m	dimensionless time the increase in pressure
τ_s^0	dimensionless time of braking at constant pressure
τ_s	dimensionless braking time
ζ	dimensionless spatial coordinate

Subscripts

1	the upper semi-space,
2	the bottom semi-space.

nonlinear one-dimensional heat problem of friction at braking with time-dependent pressure and sliding speed is used in this article.

The values of temperature obtained on the basis of the method of lines have been compared with the results found by the method of successive approximations from article [10].

2. Statement of the problem

Let two semi-spaces be compressed at infinity with a time-dependent pressure [13,14]:

$$p(t) = p_0 p^*(t),\, p^*(t) = 1 - e^{\frac{t}{t_m}}, 0 \le t \le t_s, \tag{1}$$

in a direction parallel to the z-axis of a Cartesian coordinate system $Oxyz$ (Fig. 1). In the initial time moment $t = 0$ the semi-spaces begin the relative sliding in the positive direction of y-axis with a speed [15]:

$$V(t) = V_0 V^*(t),\, V^*(t) = 1 - \frac{t}{t_m} - \frac{t_m}{t_s^0} p^*(t),\ 0 \le t \le t_s, \tag{2}$$

where the braking time t_s we find from the stop condition $V(t_s) = 0$. In the limiting case $t_m \to \infty$ from Eqs. (1) and (2), it follows that:

$$p^*(t) = 1,\, V^*(t) = 1 - \frac{t}{t_s^0},\ \ 0 \le t \le t_s^0, \tag{3}$$

where t_s^0 is the duration of braking at constant pressure and sliding with constant deceleration.

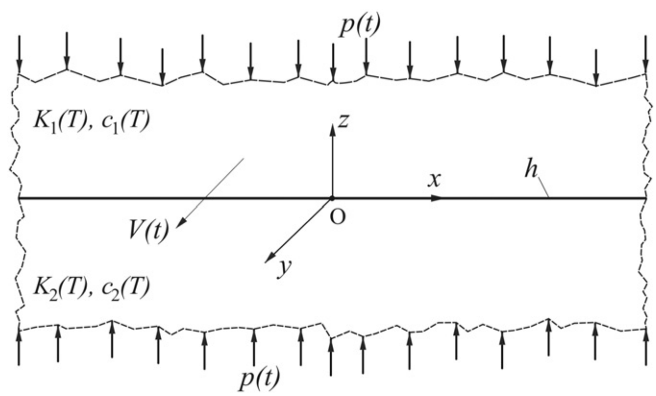

Fig. 1. Scheme of the problem.

On the contact surface $z = 0$, due to the friction, the heat is generated and the bodies are heated. It is assumed that:

1) the sum of the intensities of heat fluxes directed along the normal to the surface of contact inside the semi-spaces is equal to the specific power of friction $q(t) = q_0 q^*(t)$, where [16]

$$q_0 = f V_0 p_0,\, q^*(t) = p^*(t) V^*(t),\ 0 \le t \le t_s; \tag{4}$$

2) the thermal contact of the bodies is imperfect — through the contact surface the heat transfer takes place with a constant coefficient of thermal conductivity of contact h [17,18];
3) the coefficients of heat conduction K_l and specific heat c_l of materials are temperature dependent:

$$K_l(T) = K_{l,0} K_l^*(T), c_l(T) = c_{l,0} c_l^*(T), \tag{5}$$

where

$$K_{l,0} \equiv K_l(T_0), c_{l,0} \equiv c_l(T_0), \tag{6}$$

$K_l^*(T)$, $c_l^*(T)$, and $l = 1, 2$ are the dimensionless functions of temperature T and T_0 is the initial temperature of the semi-spaces;
4) the densities of materials ρ_l, $l = 1, 2$ are constant.

Here and further all values referring to the upper and lower semi-spaces will have subscripts 1 and 2, respectively.

Taking into account the assumptions mentioned above, we find the distribution of transient temperature field $T(z, t)$, $l = 1, 2$ in semi-spaces from solution of the following nonlinear boundary-value heat conduction problem:

$$\frac{\partial}{\partial \zeta}\left[K_1^*(T^*)\frac{\partial T^*}{\partial \zeta}\right] = \frac{c_1^*(T^*)}{k_0^*}\frac{\partial T^*}{\partial \tau}, \quad \zeta > 0,\ \ 0 < \tau \le \tau_s, \tag{7}$$

$$\frac{\partial}{\partial \zeta}\left[K_2^*(T^*)\frac{\partial T^*}{\partial \zeta}\right] = c_2^*(T^*)\frac{\partial T^*}{\partial \tau}, \quad \zeta < 0,\, 0 < \tau \le \tau_s, \tag{8}$$

$$K_2^*(T^*)\left.\frac{\partial T^*}{\partial \zeta}\right|_{\zeta = 0^-} - K_0^*\ K_1^*(T^*)\left.\frac{\partial T^*}{\partial \zeta}\right|_{\zeta = 0^+} = q^*(\tau), 0 < \tau \le \tau_s, \tag{9}$$

$$K_2^*(T^*)\left.\frac{\partial T^*}{\partial \zeta}\right|_{\zeta = 0} + K_0^* K_1^*(T^*)\left.\frac{\partial T^*}{\partial \zeta}\right|_{\zeta = 0} = Bi\left[T^*\left(0^+, \tau\right) - T^*(0^-, \tau)\right],\ \ 0 < \tau \le \tau_s, \tag{10}$$

$$\left.\frac{\partial T^*(\zeta, \tau)}{\partial \zeta}\right|_{|\zeta| \to \infty} \to 0, 0 < \tau \le \tau_s, l = 1,\ \ 2, \tag{11}$$

$$T^*(\zeta, 0) = T_0^*, \quad |\zeta| < \infty, l = 1, 2, \tag{12}$$