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# Analysis of the bioheat transfer problem with pulse boundary heat flux using a generalized dual-phase-lag model ☆

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ABSTRACT

The present work that extends the study in the literature to that laser irradiation is highly absorbed in the biological tissue and analyzes the problem based on the generalized dual-phase-lag model. A hybrid application of the Laplace transform and the modified discretization technique are used to solve the generalized dual-phase-lag model of bioheat transfer with the pulse boundary heat flux. The effects of the coupling factor between blood and tissue, porosity, and the phase lag times on the results are investigated. Comparison between the present results and the results in the literature is made and exposes some interesting phenomena. Results show that the generalized dual-phase-lag model has different temperature evolution from the classical DPL model and the Pennes equation and cannot reduce to the Pennes bio-heat transfer equation for $\tau_q = \tau_T$, even $\tau_q = \tau_T = 0$ s.


## 1. Introduction

For further improving the thermal treatment methods, bio-heat models are essential during development of equipment, for pre-planning purposes, for on-line monitoring and decision support as well as for evaluation of the extent of thermal damage [1]. Many models have been proposed by researchers to mathematically describe the complex thermal behavior of the human body. Antaki [2] used the dual-phase-lag (DPL) model, which solves the paradox that occurred in the Fourier law and introduces a phase lag for temperature gradient absent in the thermal wave model in order to account the effect of micro-structural interactions [3], to interpret heat conduction in processed meat that was interpreted with the thermal wave model. Xu et al. [4,5] presented a system discussion to the DPL effects on the biothermomechanical behavior of the skin tissue. The DPL model of bio-heat transfer has been used in theoretical studies [6–9] but equilibrium heat transfer in living tissue was considered.

In substance, the biological tissue can be split into vascular region (blood vessel) and extravascular region (tissue). The whole anatomical structure can be treated as a fluid saturated porous medium [10]. Heat transfer in living biological tissue was also considered as non-equilibrium heat transfer [10–12]. Nakayama and Kuwahara [10] and Xuan and Roetzel [11] developed the two-temperature models based on volume average to the local instantaneous governing equation for blood and tissue. The blood temperature differs from the tissue temperature, and the blood temperature varies due to convective heat transfer between the blood and tissue and blood perfusion. Zhang [12] further derived the generalized dual-phase-lag bioheat transfer equation based on the two-temperature model. In the generalized dual-phase-lag bioheat transfer equation, the phase lag times can be calculated with the properties of blood and tissue, interphase convective heat transfer coefficient and blood perfusion rate. Afrin et. al. [13] explored thermal response in tissue based on the generalized dual-phase-lag bioheat transfer equation. Narasimhan and Sadasivam [14] also modeled the thermal behavior of the human eye during retinal laser irradiation.

The present work solves the generalized dual-phase-lag bioheat transfer equation with the pulse heat flux boundary condition for the problem that laser irradiation is highly absorbed in the tissue. There are mathematical difficulties for solving such a problem [9]. For accurate and stable solutions, an efficient analytical method is required. The hybrid numerical scheme [9] based on Laplace transform is employed to solve the problem. For convenience of analysis, the literature [13,14] regards the phase lag times as the independent parameters. According to [12], the phase lag times depend on the porosity, heat capacities of blood and tissues, coupling factor, and the ratio of thermal conductivity of tissue and blood. The present work notices this character and takes it into account. A discussion to the discrepancy of the present results with those in the literature [13] and the results based on the classical dual-phase-lag bioheat transfer equation is done. At the same time, the present work would inspect the conclusion in the literature [13] that the generalized dual-phase-lag bioheat transfer equation reduces to the Pennes bio-heat transfer equation for $\tau_q = \tau_T = 0$ s.

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**Nomenclature**

| | |
|---|---|
| $c$ | specific heat of tissue, J/kg·K |
| $c_b$ | specific heat of blood, J/kg·K |
| $f$ | parameter defined in Eq. (19) |
| $G$ | coupling factor between blood and tissue |
| $H$ | new dependent variable, $H = T - T_b$ |
| $\widetilde{H}$ | Laplace transform of $H$ |
| $k$ | thermal conductivity, W/m·K |
| $\ell$ | distance between two neighboring nodes, m |
| $n$ | total number of nodes |
| $q_0$ | laser density, W/m$^3$ |
| $q_m$ | metabolic heat generation, W/m$^3$ |
| $q_r$ | spatial heating source, W/m$^3$ |
| $R_d$ | diffuse reflectance of light at the irradiated surface |
| $s$ | Laplace transform parameter |
| $t$ | time, sec |
| $T$ | temperature of tissue, °C |
| $T_b$ | arterial temperature, °C |
| $T_i$ | initial temperature of tissue, °C |
| $w_b$ | perfusion rate of blood, m$^3$/s/m$^3$ |

*Greek symbols*

| | |
|---|---|
| $\varepsilon$ | porosity |
| $\lambda$ | parameter defined in Eq. (19) |
| $\rho$ | density, Kg/m$^3$ |
| $\tau_q$ | phase lag of the heat flux |
| $\tau_T$ | phase lag of the temperature gradient |
| $\phi_{in}$ | incident laser irradiance |

*Subscripts*

| | |
|---|---|
| $B$ | blood |
| $i$ | node number |
| $j$ | number of sub-space domain |

## 2. Problem formulation

Based on equilibrium heat transfer assumption, the classical dual-phase-lag model of bioheat transfer is written as:

$$\left(1+\tau_T\frac{\partial}{\partial t}\right)k\nabla^2 T=\left(1+\tau_q\frac{\partial}{\partial t}\right)\left[\rho c\frac{\partial T}{\partial t}-w_b\rho_b c_b(T_b-T)-q_m-q_r\right] \quad (1)$$

where $k$, $\rho$, $c$, and $T$ denote the conductivity, density, specific heat, and temperature of tissue. $t$ is the time. $c_b$ and $w_b$ are, respectively, the specific heat and perfusion rate of blood. $q_m$ is the metabolic heat generation and $q_r$ is the heat source for spatial heating. $T_b$ is the arterial temperature. $\tau_q$ means the phase lag of the heat flux and $\tau_T$ means the phase lag of the temperature gradient.

However, Nakayama and Kuwahara [10] recognized that the temperatures of blood and tissue are different and the equilibrium heat transfer assumption is invalid for heat transfer in living biological tissue and developed a two-temperature model for bioheat transfer and blood flow as

$$\varepsilon\rho_b c_b\left[\frac{\partial T_b}{\partial t}+V\cdot\nabla T_b\right]=\varepsilon k_b\nabla^2 T_b+G(T-T_b)+\varepsilon q_r \quad (2)$$

$$(1-\varepsilon)\rho c\frac{\partial T}{\partial t}=(1-\varepsilon)k\nabla^2 T+G(T_b-T)+(1-\varepsilon)q_m+(1-\varepsilon)q_r \quad (3)$$

where $G = a_b h_b + w_b c_b$ is the coupling factor between blood and tissue. $h_b$ and $a_b$ are the convective heat transfer coefficient and the specific area of the blood vessel in the tissue, respectively. $\varepsilon$ is a proportional rate and subscript $b$ is referred to blood.

The generalized dual-phase-lag bioheat transfer equation with tissue temperature as sole unknown was derived by Zhang [12] based on the two-temperature model and can be written as

$$\left(1+\tau_q\frac{\partial}{\partial t}\right)(\rho c)_{eff}\frac{\partial T}{\partial t}=\left(1+\tau_T\frac{\partial}{\partial t}\right)k_{eff}\nabla^2 T+G(T_b-T)+\left(1+\frac{\varepsilon\rho_b c_b}{G}\frac{\partial}{\partial t}\right)[(1-\varepsilon)q_m+q_r] \quad (4)$$

where

$$(\rho c)_{eff}=\varepsilon\rho_b c_b+(1-\varepsilon)\rho c \quad (5)$$

$$k_{eff}=\varepsilon k_b+(1-\varepsilon)k \quad (6)$$

and

$$\alpha_{eff}=\frac{k_{eff}}{(\rho c)_{eff}} \quad (7)$$

are effective heat capacity, thermal conductivity, and thermal diffusivity, respectively. This equation includes the effects from the blood flow, thermal diffusion and the local thermal non-equilibrium between the blood and the peripheral tissue. The phase lag times depend on the porosity, heat capacities of blood and tissues, coupling factor, and the ratio of thermal conductivities of tissue and blood. The phase lag times $\tau_q$ and $\tau_T$ can be obtained with

$$\tau_q=\frac{\varepsilon(1-\varepsilon)}{[\varepsilon/C_{sb}+(1-\varepsilon)]}\frac{\rho_b c_b}{G} \quad (8)$$

$$\tau_T=\frac{\varepsilon(1-\varepsilon)}{[\varepsilon/K_{sb}+(1-\varepsilon)]}\frac{\rho_b c_b}{G} \quad (9)$$

where $C_{sb}=\rho c/(\rho_b c_b)$ and $K_{sb}=k/k_b$.

Consider a broad laser beam with a uniform irradiance ($\phi_{in}$) is applied normally to a finite slab of biological tissue with a thickness of $L$ at time $t = 0^+$. When the spot size of the broad laser beam is much larger than the thickness of the thermally affected zone for the time period of interest, a 1-D model would be sufficient for analyzing the thermal response of the heated medium [13,15].

The 1-D form of Eq. (4) with constant thermal parameters is written as

$$\left(1+\tau_q\frac{\partial}{\partial t}\right)(\rho c)_{eff}\frac{\partial T}{\partial t}=\left(1+\tau_T\frac{\partial}{\partial t}\right)k_{eff}\frac{\partial^2 T}{\partial x^2}+G(T_b-T)+\left(1+\frac{\varepsilon\rho_b c_b}{G}\frac{\partial}{\partial t}\right)[(1-\varepsilon)q_m+q_r]. \quad (10)$$

The studied problem has the initial conditions

$$T(x,0)=T_b \text{ and } \frac{\partial T(x,0)}{\partial t}=0. \quad (11)$$

When laser irradiation is highly absorbed in the tissue as for some UV and IR wavelengths, scattering may be neglected. For convenience of comparison, this work regards the transport behavior of laser light in the tissue as highly absorbed. This study assumes the laser irradiance $\phi_{in}$ is deposited as

$$\phi_{in}=q_0[u(t)-u(t-\Delta t)] \quad (12)$$

where $u(t)$ is the unit step function, $\varphi_{in}$ is the incident laser irradiance, $q_0$ the laser intensity, and $\Delta t$ the laser exposure time.