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Constructal optimization of cylindrical heat sources surrounded with a fin based on minimization of hot spot temperature☆

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ABSTRACT

Based on constructal theory, the construct of a three-dimensional cylindrical heat sources with convection heat transfer is investigated in this paper. The heat source model is composed of several cylindrical heat sources surrounded with a cylindrical fin or a conical fin. The heat source model not only exists in the heat generating electronic component surrounded with a fin to enhance heat transfer, but also in the heat generating fuel rod with zircaloy cladding. For the specified volumes of the fin and the heat sources, this construct is optimized by taking the minimization of the dimensionless hot spot temperature as optimization objective. The results show that the hot spot temperature of the model is effectively reduced after constructal optimization, which will provider some theoretical guidelines for the optimal design of heat source system.

1. Introduction

As the microelectronic and micromachining technology advances, the characteristic of electronic devices are trending towards miniaturization, high integration and high work frequency, which brings a surge of the cooling load for electronic devices and their integrated system. However, some traditional cooling designs and methods are reaching the physical limits and electronic cooling technology has become one of the technical bottlenecks, which severely restricts the development of electronics, information and control technology.

Because the performance of electronic equipment has a direct impact on its temperature, it is important to keep the hot spot temperature within acceptable temperature levels. To cope with this issue there are several methods recently proposed [1–25], such as inserting high conductivity channels in a heat generating body [1–12] and adopting different cooling flows over a heat generating body [13–25], etc. Furthermore, optimal arrangement of heat sources is an important approach to enhance the heat transfer. Arranging electronic components reasonably can increase the heat transfer rate of the system. Bejan's Constructal theory [26–30] has been widely used in heat transfer optimization since it was established and applied to the heat conduction optimization [1]. References [31–51] reflect the main research progresses in the aspect of constructal optimization of heat sources.

Some scholars have studied constructal optimization of heat sources in the two-dimensional conditions. Taking the maximum thermal conductance as an objective, Ledezma *et al.* [40] optimized the spacing of the pin fin with impinging flow, and analyzed the influences of the Prandtl number and Reynolds number on the optimal fin spacing. Flower *et al.* [41] further studied the optimal spacing between staggered parallel plates in a fixed volume with forced convection heat transfer, and maximized the total heat transfer rate, which laid a foundation to the studies on the optimal distribution of body heat sources in a fixed volume. Bejan *et al.* [42] and Stanescu *et al.* [43] reported the optimal spacing of horizontal cylinders in a fixed volume in natural convection and forced convection respectively, by taking the heat transfer density and overall thermal conductance as objectives. Bello-Ochende and Bejan [44,45] further considered the constructal multi-scale cylinders with natural convection and forced convection, and obtained the maximization of the heat transfer rate. Page *et al.* [46] studied the thermal behavior of an assembly of rotating cylinders cooled by natural convection with the objective of maximizing the heat transfer density. They also analyzed the influence of Rayleigh number and cylinder rotation speed on the optimal spacing. It was found that the optimal spacing decreases and the heat transfer density increases as the Rayleigh number or the cylinder rotation speed increase. Joucaviel *et al.* [47] optimized the spacing between the consecutive rotating cylinders, and obtained the maximum heat transfer density of the cylinders. Page *et al.* [48] investigated the thermal behavior of an assembly of multi-scale rotating cylinders cooled by natural convection, and obtained the maximum heat transfer density. Mobedi and Sunden [49] explored the influence of the thermal heat source's location in a vertical plate fin on the natural convection heat transfer. The result showed: There exists an optimal heat source

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Nomenclature

b	ratio of the center-to-center distance of the fin and heat source to the radius of fin
e	center-to-center distance of the fin and the heat source
f	radius ratio of the top circle to the lower circle
H	height
h	convective heat-transfer coefficient
k	thermal conductivity
n	number of the heat sources
q'''	heat generating rate
R	radius
T	temperature
T_∞	environment temperature
V	volume
x,y,z	cartesian coordinates

Greek symbols

ϕ	volume fraction occupied by the heat sources

Subscripts

f	fin
m	minimum
max	hot spot temperature
opt	optimum
s	heat sources
1	cylindrical fin
2	heat sources
3	top circle
4	lower circle

Superscripts

(~)	dimensionless variables

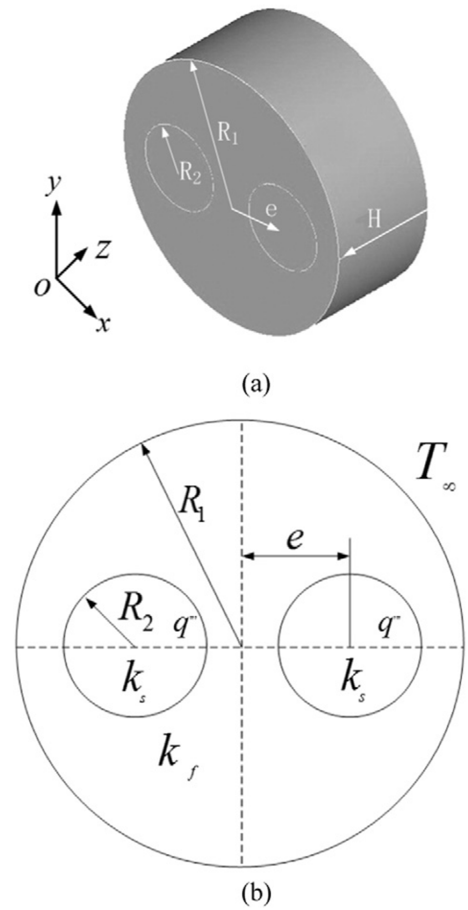

Fig. 1. Two-heat-source model.

location corresponding to the maximum heat transfer rate for fins with large conduction-convection coefficient, while the location of heat source has little influence for fins with a small conduction-convection coefficient. Hajmohammadi *et al.* [50,51] applied constructal design to the optimization of the multiple heat sources surrounded with circular- and square-shaped fins, respectively, and minimized the hot spot temperatures of the heat sources. They concluded that for the two heat source systems, the triangular distribution of the heat sources was superior to the in-row distribution.

The two-dimensional model of the heat generating body surrounded with a fin, in Refs. [50,51], has shown its advantage in improving the heat transfer performance of the heat source. This model not only has its application in the cooling of electronic components (the cylindrical electronic component surrounded with a metal fin), but also that of the heat generating fuel rods (the cylindrical fuel rod with zircaloy cladding). The cooling of heat generating fuel rods with high heat density is a big concern in nuclear power plants, which is related to the stability and the safety of the whole nuclear system. Since the practical heat generating body is three-dimensional one, based on the two-dimensional model in Ref. [50], the construct of a three-dimensional cylindrical heat source surrounded with a fin will be optimized by taking the minimization of the dimensionless hot spot temperature as optimization objective in this paper. The effects of the height and the number of the heat sources on the optimal construct of the model will be analyzed.

2. Model of three-dimensional cylindrical heat sources

The three-dimensional model of cylindrical heat sources is shown in Figs. 1(a) and 2(a). The radius of the heat sources is R_2 and the height is H. Heat is generated with a uniform rate q'''. To enhance the heat dissipation, the heat sources are connected to and cooled by a cylindrical fin with a radius R_1 and a height H. The convective heat-transfer coefficient, h, is considered to be uniform over the lateral surfaces, and the upper and lower surfaces are assumed to be adiabatic. The environment temperature is T_∞. The materials of the heat sources and the fin are both isotropic, with the thermal conductivity k_s and k_f respectively, where $k_f > k_s$. The thermal conductivities (k_s and k_f) of the heat sources and the fin are assumed to be independent of the temperature or position. Heat sources are symmetrically distributed in the fin and the number of which is n. The end views of the two-heat-source model ($n = 2$) and three-heat-source model ($n = 3$) are shown in the Figs. 1(b) and 2(b), respectively. The center-to-center distance of the fin and the heat source is e, and the ratio of this distance to the radius of the fin is defined as $b = e/R_1$. The total volume occupied by the body, $V = \pi R_1^{\,2} H$, and the total volume occupied by the heat sources, $V_s = n\pi R_2^{\,2} H$, are both fixed for a uniform cross-section. The foregoing constraints can be expressed by the following relation:

$$\phi = V_s/V = R_1^{\,2}/\left(nR_2^{\,2}\right) = Const \tag{1}$$

where ϕ is the volume fraction of the heat sources. When V and V_s are fixed, ϕ is also fixed, and the radius of the body can be given as: $R_2 = R_1/(n\phi)^{1/2}$.

The heat conduction differential equation for cylindrical heat source is:

$$k_s\left(\frac{\partial^2 T}{\partial^2 x^2} + \frac{\partial^2 T}{\partial^2 y^2} + \frac{\partial^2 T}{\partial^2 z^2}\right) + q''' = 0 \tag{2}$$