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Radiative heat transfer in a parallelogram shaped cavity☆

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ABSTRACT

An exact analytical description of the internal radiative field inside an emitting-absorbing grey semi-transparent medium enclosed in a two-dimensional parallelogram cavity is proposed. The expressions of the incident radiation and the radiative flux field are angularly and spatially discretized with a double Gauss quadrature, and the temperature field is obtained by using an iterative process. Some numerical solutions are tabulated and graphically presented as the benchmark solutions. Temperature and two components of the radiative flux are finally sketched on the whole domain. It is shown that the proposed method gives perfectly smooth results.

1. Introduction

Radiative effects are important in a large class of coupled thermal problems. This has led to the development of several numerical techniques to solve the radiative transfer equation in complex geometries. The particular case of the parallelogram shaped cavity has some interesting applications in buildings and solar energy systems [1]. Natural convection studies [2–4] have been conducted in such geometries and have shown the influence of the angle between adjacent boundaries on the flow pattern and heat transfer. Magnetic effects on the convection have also been studied and large modification of the flow structure has been observed [5]. The effect of radiative transfer due to emitting-reflecting surfaces has been considered by using the radiosity technique [6]. Baïri et al. [6] note that in such a cavity, the presence of radiation incoming from the surfaces strongly affects the natural convection and may reduce it substantially for particular angles. To the best of our knowledge, radiative transfer when the medium in the cavity is participating has not been reported yet. The main goal of this paper is therefore to present accurate benchmark results which can be used to validate the results obtained by numerical methods. In the present paper we completely describe the radiative field inside a semi-transparent medium bounded by a parallelogram shaped cavity in a partially analytic way by keeping a hybrid formulation combining space and angular integrals as in Ref. [7]. The numerical treatment combines a discretization of the useful integrals and an iterative scheme to compute the temperature field at radiative equilibrium.

In the following, we first develop in Section 2 the exact expressions of the radiative source and flux filed when using a hybrid formulation combining spatial and angular integrals. We then describe the angular and spatial discretizations of the useful integrals. Finally we present some numerical results in Section 3 and end this work by a short conclusion.

2. Mathematical formulation

One considers an infinite parallelepiped with a parallelogram section, filled with an absorbing-emitting but non scattering semi-transparent grey medium, of absorption coefficient κ and unit refractive index at radiative equilibrium. The boundary surfaces are assumed isothermal with imposed temperatures and black for sake of simplicity, while the optical constants of the grey medium are not depending on the internal temperature field. The parallelogram section is divided into $N_x \times N_y$ isothermal parallelogram cells of equal lengths $\Delta x = \frac{H_x}{N_x}$ and $\Delta y = \frac{H_y}{N_y}$, where H_x and H_y are the two characteristic lengths of the parallelogram section, each of one labelled $(i,\ j)$, with $(i,\ j) \in \{1, \ldots, N_x\} \times \{1, \ldots, N_y\}$. The only considered energy transfer is radiation, whence the internal temperature field inside the parallelepiped is determined from the radiative equilibrium condition. The incident radiation and the radiative flux at a given internal point are given by:

$$G = \int_{\Omega=4\pi} I_{ij}\left(\vec{\Omega}\right) d\Omega$$
$$\vec{q_r} = \int_{\Omega=4\pi} I_{ij}\left(\vec{\Omega}\right) \vec{\Omega} d\Omega \tag{1}$$

$I_{ij}(\vec{\Omega})$ is the radiative intensity at the centre M_{ij} of the cell labelled $(i,\ j)$ for a given direction of propagation $\vec{\Omega}$. For semi-transparent grey media with a constant unit refractive index, the radiative source is simply the divergence of the radiative flux, expressed by:

$$S = \kappa\left(4\sigma T^4 - G\right). \tag{2}$$

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Nomenclature

Bis_n, Cis_n,	Altaç angular integrated Bickley–Naylor functions
$(\vec{e_x}, \vec{e_y}, \vec{e_z})$	unit vectors of the x, y, z directions
G	volumic incident radiation (Wm^{-3})
H_x	length of the cavity sides along the x direction (m)
H_y	length of the cavity sides along the y direction (m)
(i, j)	internal cells numbering
$I_{ij}(\vec{\Omega})$	intensity at the (i, j) cell centre ($Wm^{-2}Sr^{-1}$)
Ki_n	Bickley–Naylor functions
N_x	cells number on the sides parallel to the x direction
N_y	cells number on the sides parallel to the y direction
$\vec{q}_{ij}$	radiative flux vector at the (i, j) cell centre (Wm^{-2})
q^x	x-component of the radiative flux (Wm^{-2})
q^y	y-component of the radiative flux (Wm^{-2})
S	volumic radiative source (Wm^{-3})
T	temperature (K)
x, y, z	coordinate axis directions

Greek letters

Δx	characteristic cell length along the x direction (m)
Δy	characteristic cell length along the y direction (m)
κ	absorption coefficient (m^{-1})
σ	Stephan–Boltzmann constant (5.6710^{-8} Wm^{-2} K^{-4})
τ	optical depth
φ, θ	angular description of the unit vector $\vec{\Omega}$
$\vec{\Omega}$	unit vector of radiation propagation

Subscripts (superscripts)

E, N, O, S	east, north, west and south

The temperature field at radiative equilibrium is then deduced from the incident radiation field.

Let us consider $(\vec{e_x}, \vec{e_y})$ the orthogonal basis of the parallelogram section, and $\vec{e_z}$ the unit vector orthogonal to $(\vec{e_x}, \vec{e_y})$. We note φ the angle between the projection of a luminous ray on the parallelogram section and the unit vector $\vec{e_x}$, as illustrated in Fig. 1, while θ denotes the angle between the luminous ray and the unit vector $\vec{e_z}$ perpendicular to the figure's plane.

Since the parallelepiped is infinite in the $\vec{e_z}$ direction, the temperature field so as the radiative field do not depend on the z coordinate,

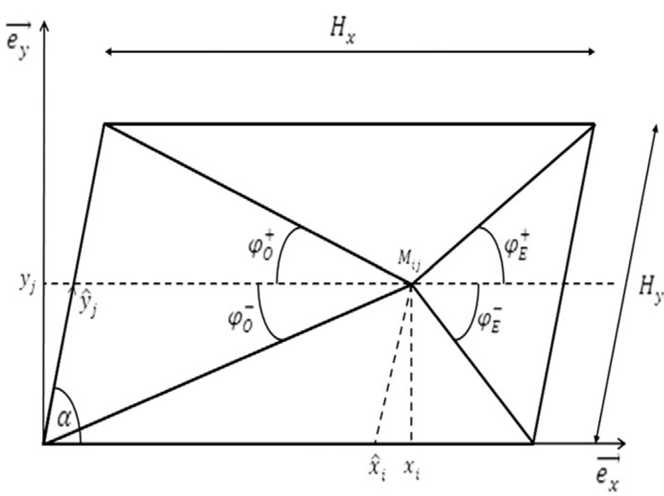

Fig. 1. Geometry of the parallelogrammic cavity.

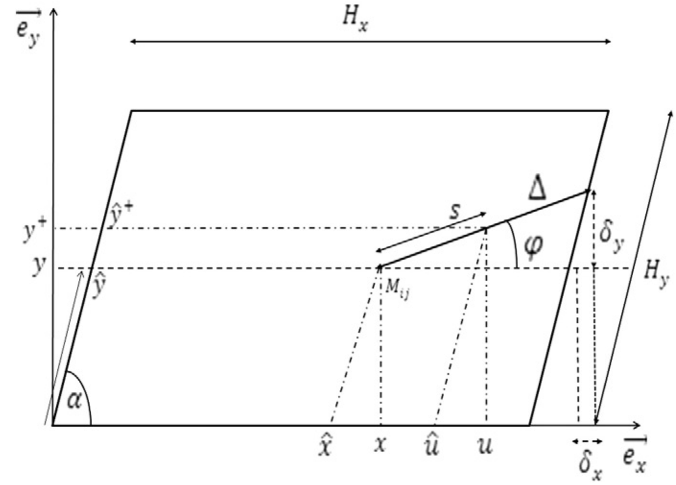

Fig. 2. Determination of the geometrical elements for radiation incoming from the northern part of the eastern boundary.

and the contribution for angles $\theta \in [0, \frac{\pi}{2}]$ is strictly equivalent to the one for angles $\theta \in [\frac{\pi}{2}, \pi]$, whence the incident radiation writes in these conditions:

$$G = \int_{\theta=0}^{\pi} \int_{\varphi=0}^{2\pi} I_{ij}(\theta, \varphi) \sin\theta \, d\theta \, d\varphi = 2 \int_{\theta=0}^{\frac{\pi}{2}} \int_{\varphi=0}^{2\pi} I_{ij}(\theta, \varphi) \sin\theta \, d\theta \, d\varphi. \quad (3)$$

We define four angular sectors in the plane $(\vec{e_x}, \vec{e_y})$ from the point M_{ij} delimiting the radiative contributions originating from the cavity's surfaces. In Eq. (3) the angle φ is the natural angle defining the propagation direction $\vec{\Omega} = \begin{pmatrix} \cos\varphi \sin\theta \\ \sin\varphi \sin\theta \\ \cos\theta \end{pmatrix}$ in the natural basis $(\vec{e_x}, \vec{e_y}, \vec{e_z})$, passing through a given point M_{ij}. This means for example that given a particular angle $\varphi \in [0, \varphi_E^+]$ where the geometrical angular sector aperture φ_E^+ is represented in Fig. 1, the radiation incoming at point M_{ij} for this direction is originating from the southern surface and reaches the eastern surface, and is not coming from the eastern one. Similarly, given an angle $\varphi \in [2\pi - \varphi_E^-, 2\pi] \equiv [-\varphi_E^-, 0]$, where the absolute value angular aperture φ_E^- is shown in Fig. 1, the incoming radiation is originating from the northern surface. We shall therefore use a ray tracing process

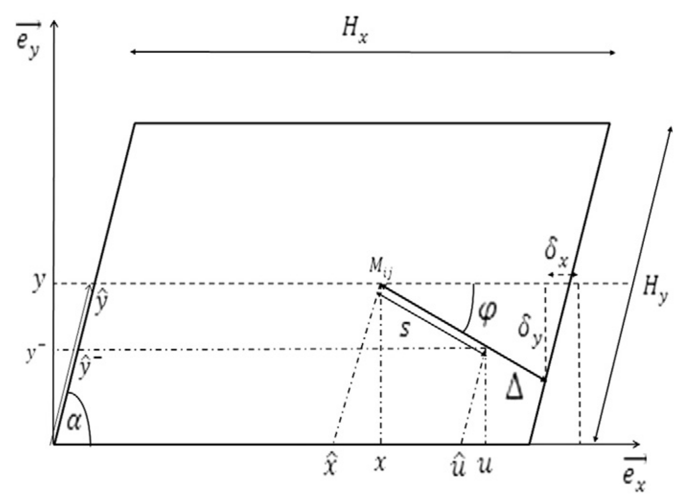

Fig. 3. Determination of the geometrical elements for radiation incoming from the southern part of the eastern boundary.