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Wall heat recirculation and exergy preservation in flow through a small tube with thin heat source ☆

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ABSTRACT

Theoretical studies have been made on heat transfer and exergy analysis of flow through a narrow tube with heat recirculating wall and embedded thin heat source. Heat transfer analysis is based on numerical solution of conservation equations of mass, momentum and energy, while the exergy analysis is based on flow exergy balance and entropy transport equation. It has been observed that the ratio of heat recirculation to heat loss (Q_R/Q_L) increases with increase in the ratio of thermal conductivity of solid wall to that of working fluid (k_s/k_g) and Peclet number of flow (Pe), while it decreases with an increase in external Nusselt number (Nu_E). The ratio Q_R/Q_L has a maximum with the ratio of wall thickness to tube radius (t_w/R). The optimum value of t_w/R depends only on k_s/k_g and reduces from a value of 0.2 at $k_s/k_g = 330$ to a value of 0.125 at $k_s/k_g = 850$, and then remains almost constant for any further increase in k_s/k_g. The volumetric entropy generation rate in the fluid flow reaches a maximum at a radial location close to the inner surface of the wall, while the entropy generation in solid wall, being more than that of the fluid, is radially uniform. The volumetric entropy generation is found to be confined within the upstream region of the heat source. Second law efficiency increases with a decrease in t_w/R and Nu_E, but with an increase in Pe. It remains almost constant with k_s/k_g.

1. Introduction

The rapid growth of microdevices puts thrust on the development of micro and mesoscale thermal systems including combustors and power sources. This brings about technical challenges in designing such systems because of their small length scale which adds additional complex features in the process performed by the system. The scientific issues and challenges pertaining to micro and mesoscale power generating devices in practice have been addressed in recent reviews [1–3]. In small scale thermal systems with internal heat generation, the ratio of heat loss to heat generation becomes significant due to high surface area to volume ratio. This poses great problem in effective utilization of energy generated within the system for its desired performance. Amongst different approaches for thermal management in the performance of small scale systems, heat recirculation through conducting wall has gained considerable interest due to its natural occurrence in small structures.

In the wake of growing concern for efficient utilization of natural energy resources, a system should be thermodynamically efficient in preserving the energy quality while meeting up the required performance criteria. The importance of exergy based thermodynamic analysis for the performance evaluation of thermal systems has been reported in literature [4–10]. The destruction of exergy (the quality of energy) takes place due to the generation of entropy in a natural process. The methods of thermodynamic optimization in minimizing the irreversible production of entropy in a process for improving its efficiency pertaining to thermal systems have been addressed in the form of review work in literature [11–14].

Hardly any work is found, in the context of wall heat recirculation and entropy generation in internal flows through very narrow tube with embedded heat source, which may provide primary information about the optimum operating conditions for efficient use of energy source in the operation of small scale devices. In the present paper, an analysis of conjugate forced convection heat transfer in a narrow cylindrical tube with embedded heat source has been made. An exergy analysis has also been supplemented with the heat transfer analysis. The objective of the present work is to have a primary understanding of the physical process of heat transfer and thermodynamic irreversibility in the tube and to explore the optimum operating conditions in terms of wall thickness and wall thermal conductivity for maximum heat recirculation as well as exergy preservation in consideration of heat and exergy loss from outer wall of the tube.

2. Theoretical formulation

Fig. 1(a). shows the schematic of the physical model consisting of a tube of radius R and length L with an embedded heat source at the middle of the tube. The heat source with a uniform volumetric energy generation

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Nomenclature

$\dot{A}$	Availability (W)
a	Specific flow availability (J/kg)
$\dot{E}$	Rate of entropy generation (W/K)
$\dot{e}$	Rate of volumetric entropy generation (W/m^3·K)
$\dot{e}^*\left(=\frac{\dot{e}T_iD^3}{Q_{gen}}\right)$	Non-dimensional rate of volumetric entropy generation
h	Specific enthalpy (J/kg)
k	Thermal conductivity (W/m·K)
$\dot{m}$	Mass flow rate of fluid (kg/s)
Nu_E	External Nusselt number
Pe	Peclet number
Q_R	Heat recirculation through wall (W)
Q_L	Heat loss from outer wall of tube (W)
Q_R^*	Non-dimensional heat recirculation (Q_R/Q_{gen})
Q_L^*	Non-dimensional heat loss (Q_L/Q_{gen})
$Q_{gen}(=\pi\dot{s}_oR^2L_c)$	Heat generation (W)
r	Dimensional radial coordinate
Re	Reynolds number
s	Specific entropy (J/kg·K)
$\dot{s}_0$	Volumetric energy generation rate (W/m^3)
u	Axial velocity component (m/s)
U	Fluid velocity at the inlet of tube (m/s)
v	Radial velocity component (m/s)
z	Dimensional axial coordinate (m).

Greek symbols

α	Thermal diffusivity of fluid (m^2/s)
ρ	Density of fluid (kg/m^3)
θ	Non-dimensional temperature
η_{II}	Second law efficiency.

Subscript

g	gas
i	inlet
o	outlet
s	solid
r	exergy reference state

Superscript

*	Non-dimensional variables

($\dot{s}_0$) has been considered to be confined over the entire cross section but within a length of L_c. The radius of the tube and the value of L_c are considered to be 0.5 mm. The length of the tube is taken to be $L = 20R$. It can be mentioned in this context that the dimensions of the tube are chosen so that it fall in the range of typical microcombustors [2]. Air is considered to be the fluid flowing through the tube. The flow is assumed to be steady, Newtonian, incompressible and axisymmetric. Such a small system with flow of air belong to the domain of continuum with no slip at the solid wall, but exhibits a very high ratio of heat loss to heat generation because of high ratio of surface area to volume. The conservation equations of mass, momentum and energy in dimensionless form are written in a cylindrical polar coordinate system as

$$\frac{\partial u^*}{\partial z^*}+\frac{1}{r^*}\frac{\partial(r^*v^*)}{\partial r^*}=0 \quad (1)$$

$$u^*\frac{\partial u^*}{\partial z^*}+v^*\frac{\partial u^*}{\partial r^*}=-\frac{\partial p^*}{\partial z^*}+\frac{2}{\mathrm{Re}}\left[\frac{\partial^2u^*}{\partial z^{*2}}+\frac{1}{r^*}\frac{\partial}{\partial r^*}\left(r^*\frac{\partial u^*}{\partial r^*}\right)\right] \quad (2)$$

$$u^*\frac{\partial v^*}{\partial z^*}+v^*\frac{\partial v^*}{\partial r^*}=-\frac{\partial p^*}{\partial r^*}+\frac{2}{\mathrm{Re}}\left[\frac{\partial^2v^*}{\partial z^{*2}}+\frac{1}{r^*}\frac{\partial}{\partial r^*}\left(r^*\frac{\partial v^*}{\partial r^*}\right)\right] \quad (3)$$

$$u^*\frac{\partial \theta_g}{\partial z^*}+v^*\frac{\partial \theta_g}{\partial r^*}=\frac{2}{Pe}\left[\frac{\partial^2\theta_g}{\partial z^{*2}}+\frac{1}{r^*}\frac{\partial}{\partial r^*}\left(r^*\frac{\partial \theta_g}{\partial r^*}\right)\right]+f(z^*) \quad (4)$$

where

$$f(z^*)=1 \quad \text{for} \quad \left(\frac{L^*-L_c^*}{2}\le z^*\le\frac{L^*+L_c^*}{2}\right)$$
$$=0 \quad \text{otherwise}$$

$$\frac{\partial^2\theta_s}{\partial z^{*2}}+\frac{1}{r^*}\frac{\partial}{\partial r^*}\left(r^*\frac{\partial \theta_s}{\partial r^*}\right)=0. \quad (5)$$

The non-dimensional variables used in the Eqs. (1)–(5) are given by $z^*=\frac{z}{R}; r^*=\frac{r}{R}; u^*=\frac{u}{U}; v^*=\frac{v}{U}; p^*=\frac{p}{\rho U^2}; \theta_{g,s}=\frac{T-T_i}{T_{ref}-T_i}; \mathrm{Re}=\frac{2RU}{\nu}; Pe=\frac{2RU}{\alpha}; L^*=\frac{L}{R}; L_c^*=\frac{L_c}{R}$. The reference temperature T_{ref}, used in the normalization of temperature T, is defined in relation to volumetric heat source $\dot{s}_0$ as

$$T_{ref}=T_i+\frac{\dot{s}_0L_c}{\rho Uc_p}. \quad (6)$$

The physical significance of T_{ref} implies a fluid temperature that can be raised adiabatically from a temperature of T_i by the utilization of total energy generated from the volumetric heat sources ($\dot{s}_0$).

The boundary conditions for the solution of Eqs. (1)–(5) are as follows:$u^*=1$; $v^*=0$; $\theta_g=0$ at $z^*=0$ and $0\le r^*\le 1\frac{\partial u^*}{\partial z^*}=0$; $\frac{\partial v^*}{\partial z^*}=0$; $\frac{\partial \theta_g}{\partial z^*}=0$ at $z^*=L^*$ and $0\le r^*\le 1u^*(1,z^*)=0$; $v^*(1,z^*)=0$; $\theta_g=\theta_s$; $k_g\frac{\partial\theta_g}{\partial r^*}=k_s\frac{\partial\theta_s}{\partial r^*}$ at $r^*=1$ and $0\le z^*\le L^*\frac{\partial u^*}{\partial r^*}=0$; $\frac{\partial v^*}{\partial r^*}=0$; $\frac{\partial\theta_g}{\partial r^*}=0$ at $r^*=0$ and $0\le z^*\le L^*$ $-\frac{k_s}{k_g}\frac{\partial\theta_s}{\partial r^*}=\frac{Nu_E\theta_w}{2}$ at $r^*=\frac{R+t_w}{R}$ and $0\le z^*\le L^*$where $Nu_E=\frac{2h_\infty R}{k_g}$and h_∞ is external convective heat transfer coefficient.

A finite volume method is used to discretize the continuity, momentum and energy equations in gas phase and energy equation in a solid phase. The conservation equations along with the boundary conditions are solved in a segregated manner. The segregated solver first solves the momentum equations, then solves the continuity equation, and update the pressure and velocity. The energy equations are subsequently solved to desired level of accuracy. Grid independent test is carried out to ensure the numerical results are independent of grid size. Finer grids are used near the wall and source where gradients are prominent.

2.1. Exergy analysis

The analysis is based on an exergy balance to a control volume as shown in Fig. 1(b). Therefore it can be written

$$\dot{A}_{f,o}=\dot{A}_{f,i}+\dot{A}_s-\dot{A}_L-\dot{I}\ . \quad (7)$$

$\dot{A}_{f,o}$, the rate of flow exergy out of the tube $=\dot{m}a_{f,o}$ where $\dot{m}$ is mass flow rate of air.

$a_{f,o}$, the specific flow exergy $=h_{f,o}-T_rs_{f,o}$ where $h_{f,o}=h_r+\int_{T_r}^{T_{f,o}}c_p(T)dT$; $s_{f,o}=s_r+\int_{T_r}^{T_{f,o}}\frac{c_p(T)dT}{T}$.

$\dot{A}_L$ is the availability loss associated with heat loss from outer wall of the tube, and is given by

$$\dot{A}_L=2\pi(R+t_w)\int_0^L k_s\left|\frac{\partial T_s}{\partial r}\right|_{r=t_w+R}\left(1-\frac{T_r}{T_w}\right)dz.$$