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Numerical simulation of natural convection in a slender cylinder under the influence of rotation☆

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ABSTRACT

The effect of rotation on natural convection in a cylindrical container, where the temperature gradient and the body force vectors are parallel, but perpendicular to the rotation vector is studied by means of a numerical simulation. The governing equations are solved with a finite volume method using the SIMPLEC decoupling strategy. We explore flows with Rayleigh and Coriolis numbers in the ranges $10^6 < Ra < 1.8 \times 10^6$ and $0 < \Omega < 4167$ respectively. The most notable differences found in the flow patterns observed with rotation as compared against the base natural convection flow with no rotation is that in contrast to the flow pattern that occurs in absence of rotation, the orientation of the convective cell is fixed with its axis of rotation parallel to the rotation vector. Also, we find that large enough Coriolis numbers can turn a time-dependent flow in the absence of rotation into a steady state flow and we determine the corresponding critical Coriolis numbers as function of the Rayleigh number. The effect of Rayleigh and Coriolis numbers on the total heat transfer (Nusselt number) is also described. It is expected that the present study can be useful for understanding some crystal growth processes.

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1. Introduction

The phenomenon of natural convection in a rotating frame of reference depends on the relative orientation and position of three vectors: the acceleration vector due to the body force, the rotation vector, and the temperature gradient vector. By and large, the situation that has been analyzed in more detail is when all three vectors are in the same direction (parallel or anti-parallel) since this configuration corresponds to local geophysical conditions; specifically, flows in the β plane [1]. A classical reference on this topic is reference [2]. Another situation which has received much less attention in the literature, but one which is of interest here is when the acceleration vector and the temperature gradient are parallel, but the rotation vector is perpendicular to them. Fig. 1 shows a sketch of the two sets of vector orientations just described.

The motivation for the present study stems from the observation that the quality of the optical and electronic properties of crystals can be improved if they are grown in centrifuges [3]. It is well established that the dynamics of natural convection in the molten material that solidifies to form the crystal ingot are determinant for the quality of the resulting crystal; in particular, time dependent convective flows result in striations in the crystal [4]. Also, it has been observed that if the crystal is grown in a centrifuge the Coriolis force has a major influence on the qualitative properties of the natural convective flow; specifically, that large temperature fluctuations (and most likely flow oscillations) observed in the flow of the molten material in absence of rotation may be reduced or suppressed by the Coriolis force. Pioneering experiments made to analyze the effect of the Coriolis force on the properties of crystals were reported in reference [5] and, further groundbreaking studies describing the stabilizing influence of the Coriolis force for natural convective flows occurring in the crystal growth processes were presented by Müller et al. [6], Weber et al. [7], Müller et al. [8], and others using both numerical and experimental techniques. Of particular importance in the context of the present investigation is the article by Weber et al. [7] who proposed a simplified physical model, similar to that used here, for analyzing the flow inside a rotating cylindrical container. This model approximately simulates the physical conditions found when crystals are grown in centrifuges. They reported the formation of a single non-axisymmetric convective cell and the existence of two possible flows. In the two cases, the axis of rotation of the convective cell is aligned with the rotation vector but in one of them, the fluid rotation vector is parallel to the rotation vector of the centrifuge and in the other, the two vectors are anti-parallel. A more complete model that incorporates realistic details of the physical conditions occurring in a container that is placed in a centrifuge which rotates around a vertical axis with the cylinder that contains the working fluid placed on a swinging mount such that the axis of the cylinder is aligned with the vector sum of the centrifugal fore and the terrestrial gravity was presented by Arnold et al. [9]. The model includes the consideration that the rotation vector is at an angle with respect to both the temperature gradient vector and the body force. The emphasis of the study is on

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Nomenclature

$\vec{a}$	acceleration due to body force
$\vec{g}$	acceleration of terrestrial gravity
h	height of the cylindrical container
i	unit vector in the x direction
Nu	average Nusselt number
n_θ	node number in the azimuthal direction
Pr	Prandtl number
r	radial coordinate
Ra	Rayleigh number ($=\beta a h^3 \Delta T/\alpha\nu$)
T_H	hot temperature
T_C	cold temperature
u_c	characteristic velocity ($=\sqrt{a\beta\Delta T h}$)
u_r	radial velocity
u_θ	azimuthal velocity
u_z	axial velocity
x	Cartesian coordinate
z	axial coordinate
Greek	
α	thermal diffusivity
Δr	length of discretization cell
$\Delta\theta$	length of discretization cell
Δz	length of discretization cell
ΔT	temperature difference $T_H - T_C$
γ	aspect ratio ($=h/D$)
ν	kinematic viscosity
$\hat{\rho}$	unit radial vector
ρ_o	reference density
$\hat{\theta}$	unit azimuthal vector
θ	azimuthal coordinate
ω	angular velocity
Ω	Coriolis number ($=\omega h^2/\nu$)
Ω^c	critical Coriolis number

the relative importance of the gradient acceleration driven buoyancy and natural convection. Study cases where one or the other effect dominates are described. In an attempt to systematize the description of the qualitative dynamic behavior of the flows, Fikri and Labrosse [10] identified the relevant non-dimensional numbers for a flow with the physical conditions similar to that analyzed by Arnold et al. and described the steady flows in rectangular parallelepiped crucibles.

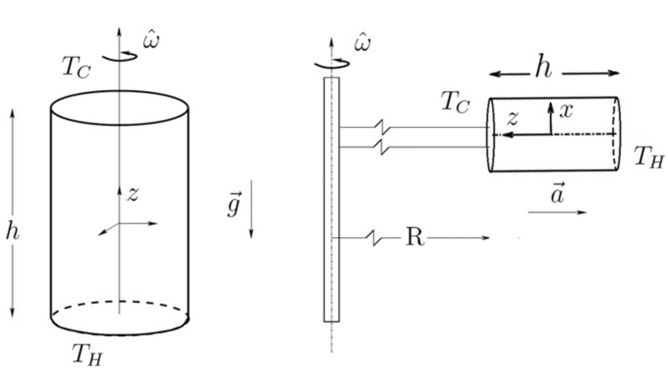

Fig. 1. Left: configuration where the acceleration due to the body force (in most of these cases this parameter is identified with the gravity acceleration $\vec{g}$), rotation and temperature gradient are parallel. Right: configuration where temperature gradient and acceleration due to the body force are parallel to each other but the rotation vector is perpendicular to these two vectors.

The present study refers to the analysis of the influence of the Coriolis force on the natural convection in a cylinder when the rotation vector is perpendicular to the temperature gradient and the body force vector with a numerical solution, emphasizing on the modifications induced in the convective pattern due to the presence of a Coriolis force. Our study may be interpreted as an approximate model of the flow prevailing on a crucible mounted on a centrifuge. Although the conditions of the situation considered in this study are only an approximation to a real flow, it is pertinent to insist that this simplified model has been used before to shed light on the different phenomena that occur in a real situation [7].

2. Governing equations

Consider the natural convective motion inside the cylindrical container of height h and diameter D with the top and bottom walls held at cold (T_C) and hot (T_H) temperatures respectively and mounted in a rotating system similar to that shown in the sketch on the right hand of Fig. 1. Observe that the only body force present is the centrifugal force which is aligned with the axis of symmetry of the cylindrical container. The flow can be described by numerically solving the mass, momentum and energy conservation equations. In many cases, the study of natural convective flows is made using the Boussinesq approximation which assumes that all physical properties of the working fluid are constant except for the density in the buoyancy term. Although the range of validity of this approximation is well established in the non-rotating case, for the rotating natural convective flow, information is much more scarce. Important comments on the applicability of the approximation in the case where all three vectors (temperature gradient, rotation and body force) are parallel can be found in reference [11]. For perpendicular rotation, the ranges of validity and physical and geometrical requirements for the application of the Boussinesq approximation are given in reference [12]. In the present study we consider that the conditions described in reference [12] are satisfied and we assume that the Boussinesq approximation can be applied.

As stated before, we assume that the rotation vector and the body force are perpendicular to each other, and since the body force vector runs along the z-axis we choose that the rotation vector points along a positive x-axis direction, i.e.

$$\hat{\omega} = \hat{i} = \cos\theta\,\hat{\rho} - \sin\theta\,\hat{\theta} \tag{1}$$

where $\hat{\omega}$ and $\hat{\imath}$ are unit vectors indicating respectively the directions of the rotation and of the x-axis. The radial direction is $\hat{\rho}$ and the azimuthal direction is denoted by $\hat{\theta}$. The conservation equations that describe the natural convection under the influence of rotation in cylindrical coordinates and using appropriate dimensionless variables are:

$$\frac{1}{r}\frac{\partial}{\partial r}(r u_r) + \frac{1}{r}\frac{\partial u_\theta}{\partial \theta} + \frac{\partial u_z}{\partial z} = 0, \tag{2}$$

$$\begin{aligned}\frac{\partial u_r}{\partial t} + \left(\vec{u}\cdot\nabla\right)u_r - \frac{u_\theta^2}{r} = -\frac{\partial p}{\partial r} + \left(\frac{Pr}{Ra}\right)^{\frac{1}{2}}\left(\nabla^2 u_r - \frac{u_r}{r^2} - \frac{2}{r^2}\frac{\partial u_\theta}{\partial \theta}\right)\\ + 2\Omega\left(\frac{Pr}{Ra}\right)^{\frac{1}{2}} u_z \sin\theta,\end{aligned} \tag{3}$$

$$\begin{aligned}\frac{\partial u_\theta}{\partial t} + \left(\vec{u}\cdot\nabla\right)u_\theta + \frac{u_\theta u_r}{r} = -\frac{1}{r}\frac{\partial p}{\partial \theta} + \left(\frac{Pr}{Ra}\right)^{\frac{1}{2}}\left(\nabla^2 u_\theta - \frac{u_\theta}{r^2} + \frac{2}{r^2}\frac{\partial u_r}{\partial \theta}\right)\\ + 2\Omega\left(\frac{Pr}{Ra}\right)^{\frac{1}{2}} u_z \cos\theta,\end{aligned} \tag{4}$$

$$\begin{aligned}\frac{\partial u_z}{\partial t} + \left(\vec{u}\cdot\nabla\right)u_z = -\frac{\partial p}{\partial z} + \left(\frac{Pr}{Ra}\right)^{\frac{1}{2}}\nabla^2 u_z\\ + T - 2\Omega\left(\frac{Pr}{Ra}\right)^{\frac{1}{2}}(u_\theta\cos\theta + u_r\sin\theta),\end{aligned} \tag{5}$$

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