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Thermal performance of continuously moving radiative–convective fin of complex cross-section with multiple nonlinearities ☆

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ABSTRACT

Spectral collocation method (SCM) is adopted to predict the temperature distribution in the fin with temperature dependent thermal conductivity, heat transfer coefficient and surface emissivity. These temperature dependent properties or parameters cause multiple nonlinearities of energy equation. In order to reduce these multiple nonlinearities, a local linearization approach is adopted. The spatial distribution of dimensionless temperature is discretized by Lagrange interpolation polynomials. Accordingly, the differential form of energy equation is transformed to the matrix form of algebraic equation. The accuracy of the SCM model is verified by comparing SCM results with available data in references. Meanwhile, compared with analytical solutions, it can be found that the convergence rate of SCM approximately follows exponential law. In addition, effects of various physical parameters, such as Peclet number, thermal conductivity parameter, emissivity parameter, parameter of heat transfer coefficient, convective–conductive parameter and radiative–conductive parameter on the dimensionless temperature, the dimensionless fin-tip temperature and the volume adjusted fin efficiency are comprehensively analyzed.

1 . Introduction

Fined surfaces are generally used as a heat dissipation mechanism to enhance heat transfer rate between primary surface and the environment in heat exchangers [1,2]. They have widely applications in engineering industries, such as low-temperature flue gas utilization systems of waste heat energy, heat exchangers in power plants, heat exchanger of heat pump, etc. Most thermal performance analyses in the fin are based on the assumptions of uniform temperature distribution along the fin cross-section and constant thermo-physical properties. These assumptions can simplify energy equation from partial difference equation to ordinary difference equation, and the analytical solution of energy equation can be obtained. However, these assumptions are inconsistent with the thermal performances in the fin under realistic operation conditions which invariably involve multiple nonlinearities. Because of these multiple nonlinearities, it is impossible to obtain the analytical solution of energy equation. Therefore, many researchers try to solve energy equation by approximation or numerical methods.

One of nonlinearities arises when thermal conductivity is varied with temperature. If a large temperature variation exists in the fin, thermal conductivity of the fin may be varied with temperature [3]. For instance, thermal conductivity of aluminum fin decreases from 302 $W \cdot m^{-1} \cdot K^{-1}$ at 100 K to 218 $W \cdot m^{-1} \cdot K^{-1}$ at 800 K; thermal conductivity of AISI 302 stainless steel fin increases from 17.3 $W \cdot m^{-1} \cdot K^{-1}$ at 400 K to 25.5 $W \cdot m^{-1} \cdot K^{-1}$ at 1000 K [3]. As early as in the middle of 1980s, Aziz and Huq [4] gave rigorous formulations by a perturbation method, and established the optimum fin parameter on temperature dependent thermal conductivity. Coskun and Atay [5] developed the variation iteration method to analyze the fin efficiency of straight convective fins with temperature dependent thermal conductivity. Kulkarni and Joglekar [6] proposed a numerical technique based on residue minimization to solve the nonlinear energy equation in straight convective fins with temperature dependent thermal conductivity. Domairry and Fazeli [7] utilized homotopy analysis method (HAM) to evaluate the fin efficiency in the straight fin with variable thermal conductivity.

Another nonlinearity appears when heat transfer coefficient is a function of temperature. In real applications, the dependence of heat transfer coefficient is usually expressed as a power law form where its power depends on heat transfer mode like laminar natural convection, turbulent natural convection, condensation and boiling. This phenomenon had been confirmed by experimental results of Sertkaya et al. [8]. Lesnic et al. [9] presented a decomposition solution in terms of the ordinary functions of heat transfer for a straight fin. In this work, heat transfer coefficient is varied as a power–law function of the temperature difference between the surface and the convective sink. Sadri et al. [10] considered a constant cross-section area with temperature dependent thermal conductivity and heat transfer coefficient, and used differential transformation method (DTM) to obtain approximated analytical solutions for the temperature distribution and the fin efficiency. Kahani

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Nomenclature

A	thermal conductivity parameter
B	emissivity parameter
b_j	coefficient of the integral term weight
C	Fin taper ratio
C_1, C_2	constants in Eq. (36)
c_p	specific heat capacity at constant pressure, $J \cdot kg^{-1} \cdot K^{-1}$
$D_{i,j}^{(1)}$	entries of the first order derivative matrix
$D_{i,j}^{(2)}$	entries of the second order derivative matrix
$F_{i,j}$	entries of spectral coefficient matrix which are defined in Eq. (23)
$G_{i,j}$	entries of spectral coefficient matrix which are defined in Eq. (28)
H_i	entries of spectral coefficient matrix which are defined in Eq. (24)
h	convective heat transfer coefficient, $W \cdot m^{-2} \cdot K^{-1}$
h_i	barycentric Lagrange interpolation polynomials
h_L	convective heat transfer coefficient corresponding to the temperature difference $T_L - T_c$, $W \cdot m^{-2} \cdot K^{-1}$
L	fin tip length, m
l_1, l_2	adjustment parameters to reduce the nonlinearity of energy equation
m	parameter of variable heat transfer coefficient
N	total number of collocation points
N_{cc}	convective–conductive parameter
N_{rc}	radiative–conductive parameter
P	perimeter, m
Pe	Peclet number
Q_i	entries of spectral coefficient matrix which are defined in Eq. (29)
q_f	fin heat transfer rate, $W \cdot m^{-1}$
q_f^*	volume adjusted heat transfer rate, $W \cdot m^{-1}$
q_{ideal}	ideal heat transfer rate, $W \cdot m^{-1}$
R_{volume}	volume ratio
s_i	Chebyshev–Gauss–Lobatto collocation points
T	temperature, K
T_c	ambient fluid temperature, K
T_L	temperature at fin base, K
T_s	radiation sink temperature, K
w_i	entries of integral matrix defined in Eq. (31)
w_j	coefficient of Lagrange interpolation polynomials
X	dimensionless axial coordinate
x	coordinate in x-direction, m

Greek symbols

α	thermal conductivity coefficient K^{-1}
β	surface emissivity coefficient, K^{-1}
δ	semi-thickness of the fin, m
δ_0	semi-fin taper thickness, m
δ_j	parameter defined in Eq. (21)
δ_L	semi-base thickness, m
ε	surface emissivity
ε_{error}	integral averaged relative error
ε_s	surface emissivity at radiation sink temperature
η	fin efficiency
η^*	volume adjustment fin efficiency
Θ	dimensionless temperature
Θ^*	the last iterative value of dimensionless temperature
Θ_c	dimensionless environment temperature
Θ_s	dimensionless radiation sink temperature
λ	thermal conductivity, $W \cdot m^{-1} \cdot K^{-1}$
λ_0	thermal conductivity at convection sink temperature, $W \cdot m^{-1} \cdot K^{-1}$
ρ	density of the fin material, $kg \cdot m^{-3}$
σ	Stefan–Boltzmann constant, $W \cdot m^{-2} \cdot K^{-4}$

Subscripts

i,j,k	solution node indexes
max	maximum value
min	minimum value

et al. [11] utilized HAM to evaluate the analytical approximate solution and the fin efficiency of the nonlinear fin problem with variable thermal conductivity and heat transfer coefficient. Mosayebidorcheh et al. [12] used DTM to solve the nonlinear heat transfer equation of the fin when both thermal conductivity and heat transfer coefficient are power–law temperature dependent. Kani and Aziz [13] developed HAM for evaluating the thermal performance of a straight trapezoidal fin with temperature dependent thermal conductivity and heat transfer coefficient.

The other nonlinearities arise in the energy conservation equation of the fin due to the Stefan–Boltzmann law for radiation and temperature dependent internal heat generation [14]. Torabi and Aziz [15] developed DTM to analyze the thermal performance and the fin efficiency of T-shape cross-section with temperature dependent thermal conductivity, heat transfer coefficient and surface emissivity. Torabi and Zhang [16] analytically investigated the temperature distribution and efficiency of convective–radiative straight fins of various cross-sections with simultaneous variation of thermal conductivity, heat transfer coefficient, surface emissivity and internal heat generation. Recently, Torabi et al. [17] comparatively studied convective–radiative fins of rectangular, trapezoidal and concave parabolic profiles with simultaneous variation of thermal conductivity, heat transfer coefficient and surface emissivity depending on temperature.

Spectral collocation method (SCM) is a high order numerical method which is based on Chebyshev polynomials [18]. In the field of numerical simulations, lower order methods, like finite volume method and finite element method, can provide linear convergence rate, while SCM can provide exponential convergence rate [19,20]. Due to the mathematical simplicity and high accuracy with relatively few spatial grid points necessary, SCM is considered to be an efficient technique in science and engineering applications, such as computational fluid dynamics [21–23], electromagnetics [24], and magneto-hydrodynamics [25–27]. Recently, Li et al. successfully developed SCM to analyze thermal radiation heat transfer [28] and coupled radiation and conduction heat transfer [29] in the semitransparent medium.

In this research, we extend SCM to solve the radiative–convective heat transfer in the moving fin of complex cross-section with variable thermal conductivity, heat transfer coefficient, and surface emissivity. In the following, the physical model and mathematical formulations will be presented in Section 2. The accuracy and convergence rate of the SCM are demonstrated by available numerical results in the literature and analytical solutions in Section 3. In addition, effects of various parameters, including Peclet number Pe, thermal conductivity parameter A, emissivity parameter B, parameter of heat transfer coefficient m, convective–conductive parameter N_{cc}, and radiative–conductive parameter N_{rc}, on the dimensionless temperature distribution, the dimensionless fin-tip temperature and the fin efficiency in the moving fin of complex cross-section are also analyzed in Section 3. Finally, conclusions are summarized in Section 4.

2 . Physical model and mathematic formulations

2.1. Physical and mathematical models

As shown in Fig. 1, we consider the thermal processing of continuously moving fins of trapezoidal, concave parabolic and convex cross-sections with perimeter P and constant speed U. Surfaces of these