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Geometry estimation for the inner surface in a furnace wall made of functionally graded materials☆

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ABSTRACT

The aim of this study is to solve an inverse geometry heat conduction problem (shape identification problem) to estimate the unknown geometry of the inner surface in a furnace wall which is made of functionally graded materials (FGMs). The inner surface geometry is estimated from the temperatures of measured points within the furnace wall. The inverse algorithm used in the study is based on the conjugate gradient method (CGM) and the discrepancy principle. The effect of measurement errors and measurement locations on the estimation accuracy is also investigated. Two different examples are discussed. Results show that the unknown geometry of the inner wall surface can be predicted precisely by using the present approach.

1. Introduction

Functionally graded materials, originally proposed by Japanese researchers [1], are nonhomogeneous materials within which physical properties vary continuously. The smooth variation of properties results from continuous transition of the volume fraction of constituents. These novel materials have excellent thermo-mechanical properties to withstand high temperature and have been extensively applied to important structures, such as nuclear reactors, pressure vessels and pipes, and chemical plants [2–4]. In recent years, FGMs have even been proposed as a solution for aerospace industry where temperature resistant, light-weight structures are required to meet the challenges faced by future high-speed space vehicles.

In the past several decades, inverse analysis has been widely applied to solve engineering problems. In the heat transfer area, external inverse problems include estimation of temperature, heat flux, or heat transfer coefficient [5,6], and internal inverse problems include determination of thermophysical properties, such as thermal conductivity and heat capacity [7,8]. In addition, the inverse analysis has also been applied to the problems related to shape design [9–11] and shape identification [12–14].

The inside of furnace experiences extremely severe and complex conditions for a long operating period in industry. Both corrosion effects and thermal stress can damage the furnace walls. Hence, monitoring the shape and temperature of inner surface is quite essential to ensure the safe operation of the furnaces. Su et al. [15] studied the shape identification problems of inner surface of furnaces, and in their work they supposed that the temperature of the inner wall was known and its distribution was uniform. Wang et al. [16] solved the temperature identification problems of furnace inner surface by a fuzzy inference method (FIM). Comparisons with the conjugate gradient method and the genetic algorithm (GA) are also conducted.

Although a great number of reports dealing with the shape identification problems of homogeneous mediums have been available, however, to the best of the authors' knowledge, the study on the shape identification problem of FGMs is limited in the literature. The aim of the present study is to develop an inverse analysis for estimating the unknown geometry of the inner surface in a furnace wall which is made of functionally graded materials, from the knowledge of temperature measurements taken within the wall. The heat conduction problem that the inverse method is applied is steady state, hence only a couple of spatial temperature measurements are needed. Here, we present the conjugate gradient method [17–19] and the discrepancy principle [20] to estimate the unknown inner surface geometry by using the simulated temperature measurements. The conjugate gradient method with an adjoint equation, also called Alifanov's iterative regularization method, belongs to a class of iterative regularization techniques, which mean the regularization procedure is performed during the iterative processes, thus the determination of optimal regularization conditions is not needed. No prior information is used in the functional form of the unknown surface geometry. On the other hand, the discrepancy principle is used to terminate the iteration process in the conjugate gradient method.

2. Analysis

2.1. Direct problem

In this work a two-dimensional furnace wall system, as shown in Fig. 1, is considered. The outer radius of the furnace wall is a constant

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Nomenclature

Bi	Biot number (hr_o/k_o)
F	unknown inner surface geometry of the furnace (m)
f	dimensionless unknown inner surface geometry of the furnace
h	convection heat transfer coefficient (W m^{-2} K^{-1})
J	functional
J'	gradient of functional
k	thermal conductivity (W m^{-1} K^{-1})
k_o	thermal conductivity at the outer surface of the furnace (W m^{-1} K^{-1})
n	nonhomogeneity index
p	direction of descent
r	radius of the furnace (m)
r_i	inner radius of the furnace (m)
r_m	radius of temperature measurement positions (m)
r_o	outer radius of the furnace (m)
(r, ϕ)	cylindrical coordinates
T	temperature (K)
T_i	temperature of inner surface of the furnace (K)
T_∞	surrounding temperature (K)

Greek symbols

Δ	small variation quality
β	step size
γ	conjugate coefficient
ε	very small value
η	dimensionless radius of the furnace
η_m	dimensionless radius of temperature measurement positions, r_m/r_o
θ	dimensionless temperature
λ	variable used in the adjoint problem
σ	uncertainty of temperature measurement
ϖ	random variable

Superscript/subscript

N	iterative number

value of r_o, and the inner radius $r_i(\phi)$ of the wall is assumed to be dependent on the polar angle ϕ. To establish an appropriate physical model, some reasonable assumptions are presented. The temperature $T(r, \phi)$ in the furnace wall is regarded as steady after a long time of operation. In addition, unlike the standard analysis, which assumes the furnace wall material to be homogeneous with uniform thermal conductivity, the present analysis models the nonhomogeneity of the furnace wall by allowing the thermal conductivity k of furnace wall to vary as a power function of radial coordinate r , that is,

$$k(r) = k_o(r/r_o)^n, \tag{1}$$

where n is a constant and k_o is the thermal conductivity of outer surface of the furnace wall. Such a power dependence of the thermal conductivity occurs in some aerospace and automotive structures [21]. With the use of Eq. (1), the governing equation for the temperature, $T(r, \phi)$, of the two-dimensional furnace wall system in steady state can be expressed as [16]:

$$\frac{1}{r}\frac{\partial}{\partial r}\left[rk(r)\frac{\partial T(r,\phi)}{\partial r}\right]+\frac{1}{r^2}\frac{\partial}{\partial \phi}\left[k(r)\frac{\partial T(r,\phi)}{\partial \phi}\right]=0, r_i\le r\le r_o, 0\le\phi\le 2\pi. \tag{2}$$

The associated boundary conditions are given as:

$$T(r,\phi) = T_i, \text{at } r = r_i = F(\phi), \tag{3}$$

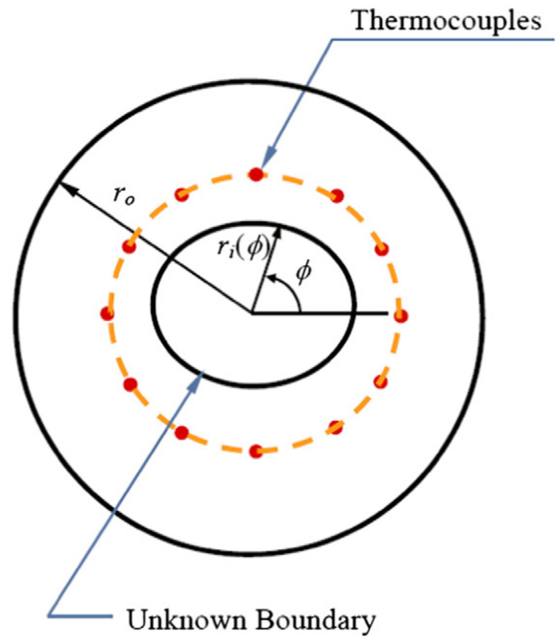

Fig. 1. Diagram for the furnace wall system.

$$-k(r)\frac{\partial T(r,\phi)}{\partial r} = h[T(r,\phi)-T_\infty], \text{ at } r = r_o, \tag{4}$$

where T_i is the temperature of inner surface of the furnace wall, h is the convective heat transfer coefficient at the outer surface, and T_∞ is the surrounding temperature. Since the temperature is periodic in ϕ with a period 2π, we can have:

$$T(r,0) = T(r,2\pi), \quad r_i\le r\le r_o, \tag{5}$$

$$\frac{\partial T(r,0)}{\partial \phi} = \frac{\partial T(r,2\pi)}{\partial \phi}, r_i\le r\le r_o. \tag{6}$$

For the convenience of numerical analysis, the following dimensionless parameters are introduced as:

$$\eta = \frac{r}{r_o}, \quad \theta = \frac{T-T_\infty}{T_i-T_\infty}, \quad f(\phi) = \frac{F(\phi)}{r_o}. \tag{7}$$

Introducing these dimensionless variables in Eq. (7) into Eqs. (1)–(6) leads to the following dimensionless equations:

$$k(\eta) = k_o\eta^n, \tag{8}$$

$$\frac{1}{\eta}\frac{\partial}{\partial \eta}\left[\eta^{n+1}\frac{\partial\theta(\eta,\phi)}{\partial\eta}\right]+\frac{1}{\eta^2}\frac{\partial}{\partial\phi}\left[\eta^n\frac{\partial\theta(\eta,\phi)}{\partial\phi}\right]=0, \tag{9}$$

$$\eta_i\le\eta\le 1, \quad 0\le\phi\le 2\pi,$$

$$\theta(\eta,\phi) = 1, \text{ at } \eta=\eta_i=f(\phi), \quad \text{at} \quad \eta=\eta_i=f(\phi), \tag{10}$$

$$-\eta^n\frac{\partial\theta(\eta,\phi)}{\partial\eta} = Bi\cdot\theta \quad \text{at} \quad \eta = 1, \tag{11}$$

$$\theta(\eta,0) = \theta(\eta,2\pi), \quad \eta_i\le\eta\le 1, \tag{12}$$

$$\frac{\partial\theta(\eta,0)}{\partial\phi} = \frac{\partial\theta(\eta,2\pi)}{\partial\phi}, \quad \eta_i\le\eta\le 1, \tag{13}$$

where the Biot number $Bi = hr_o/k_o$. The direct heat transfer problem defined by Eqs. (9)–(13) is bound for gaining the temperature field