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# Unsteady coupled thermal boundary layers induced by a fin on the partition of a differentially heated cavity☆

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ABSTRACT

The transition from steady to unsteady coupled thermal boundary layers induced by a fin on the partition of a differentially heated cavity was numerically investigated. Numerical simulations were performed over the range of Rayleigh numbers from $Ra = 10^7$ to $2 \times 10^{10}$ ($Pr = 6.63$). The temporal development and spatial structure of natural convection flows in the partitioned cavity with a fin are described. It has been demonstrated that the fin may induce a transition to unsteady coupled thermal boundary layers and the critical Rayleigh number for the occurrence of the transition is between 3.5 and $3.6 \times 10^8$. Furthermore, the peak frequency of the oscillations induced by the fin has been obtained through spectral analysis. It has been found that the flow rate through the cavity with a fin is larger than that without a fin under unsteady flow, indicating that the fin may improve unsteady flow in the partition cavity.



## 1. Introduction

Natural convection in a differentially heated cavity is extensively present in various industrial applications such as heat exchangers, and thus studies of the flow in the cavity have been extensively reported in the literature over the past decades. The study by Batchelor [1] shows that heat transfer through the cavity is dominated by conduction for sufficiently small Rayleigh numbers. Subsequent investigations (e.g. [2]) have focused on steady natural convection in the cavity. However, natural convection in industrial systems is usually unsteady. Accordingly, the development of natural convection in the cavity following sudden heating and cooling has been given considerable attention over the last three decades. The study by Patterson and Imberger [3] shows that transient natural convection flows in the cavity involve a vertical boundary layer flow, a horizontal intrusion flow, and the flow in the core. It is clear that if the Rayleigh number is sufficiently large (e.g. larger than the critical value), natural convection in the cavity could become unstable [4,5] and even fully turbulent [6,7].

Previous studies have paid more attention to natural convection in non-partitioned cavities as described above, but an increasing number of studies have also focused on natural convection in a partitioned cavity due to its extensive applications in industry. Recently, a study of transient natural convection in a partitioned cavity was performed by Xu et al. [8]. It has been demonstrated that coupled thermal boundary layers form on either side of the partition following a sudden temperature difference imposed between the fluids in the adjacent cavities, and the thicknesses of the coupled thermal boundary layers increase with time according to the scaling $O(\kappa^{1/2} t^{1/2})$. Following the sudden temperature difference, the intrusion flows from the discharges of the coupled thermal boundary layers at the respective downstream ends oscillate between the partition and sidewalls. Finally, the fluid becomes stratified around the partition. The study in [8] shows that the temperature condition on the partition changes from an initially isothermal to an approximately isoflux condition at the fully developed state. For a laminar flow, the partition, even though perfectly conducting, depresses natural convection flows and heat transfer in the cavity (see, e.g. [9–11]). The scaling relationship between heat transfer and the Rayleigh number, $Nu \sim Ra^{1/4}$, has been quantified by Anderson and Bejan [12,13]. Since the partition may depress heat transfer through the cavity, a further attempt to place additional partitions into the cavity has been carried out for thermal insulation. Experimental and numerical results in [14] show that the Nusselt number of the partitioned cavity is inversely proportional to $1 + N$, where $N$ is the number of partitions.

The above-mentioned studies show that natural convection in the partitioned cavity is steady in a fully developed stage for low Rayleigh numbers. However, the numerical and experimental results have demonstrated that as the Rayleigh number increases, fully developed flows in the partitioned cavity may be unsteady [15,16]. Williamson et al. [15] pointed out that unsteady natural convection at the fully developed stage is an absolute instability for which perturbations in the coupled thermal boundary layers on the two sides of the partition may feed each other through the partition. As the Rayleigh number increases further (e.g. $Ra > 10^{10}$) for small Prandtl numbers (e.g. $Pr = 0.71$), fully developed flows in the partitioned cavity are even turbulent [17].

Numerical simulations show that coupled thermal boundary layers around the partition are steady if the Rayleigh number is sufficiently

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**Nomenclature**

| | |
|---|---|
| $A$ | aspect ratio |
| $f$ | frequency |
| $g$ | acceleration due to gravity ($m/s^2$) |
| $H, L$ | height and length of the cavity (m) |
| $l$ | length of the fin |
| $N$ | number of partitions |
| $Nu$ | Nusselt number |
| $p$ | pressure |
| $Pr$ | Prandtl number, $\nu/\kappa$ |
| $Ra$ | Rayleigh number, $g\beta\Delta TH^3/\nu\kappa$ |
| $Q$ | volumetric flow rate |
| $t$ | time |
| $\Delta t$ | time step |
| $T$ | temperature |
| $T_c, T_h$ | temperatures of the cold and hot sidewalls (K) |
| $u, v$ | velocity components in the $x$ and $y$ directions |
| $x, y$ | horizontal and vertical coordinates |
| $\beta$ | coefficient of thermal expansion (1/K) |
| $\delta$ | thickness of the thermal boundary layer (m) |
| $\kappa$ | thermal diffusivity ($m^2/s$) |
| $\nu$ | kinematic viscosity ($m^2/s$) |
| $\rho$ | density ($kg/m^3$) |

small (e.g. $Ra < 10^{10}$ for the square partitioned cavity [5]). Clearly, this conclusion was drawn from numerical simulations without any perturbations. However, there are a variety of internal or external perturbations in an industrial system. Therefore, it is of significance to investigate coupled thermal boundary layers with perturbations. It was well known that one good way to perturb a vertical thermal boundary layer is to place a fin on the vertical wall, which is extensively present in heat exchangers [18]. For example, a number of fins are set up on the heat pipe exchanger in order to increase the heat transfer area of the pipe and in turn enhance the heat transfer through the pipe wall.

In fact, the study of the fin-induced flow has been conducted for several decades and has been considerably reported in the literature (refer to [19–21]). The focus of the early studies (e.g. [19]) is on steady laminar natural convection induced by a fin at low Rayleigh numbers. Recently, transient natural convection in the cavity with a fin has also been investigated [22–25]. The experiment by Xu et al. [22] shows that the development of natural convection flows induced by a fin following sudden heating includes three stages: an initial stage, a transitional stage, and a fully developed stage. The fin may significantly influence natural convection in the cavity and even trigger the transition to unsteady natural convection in the cavity [25–28]. That is, if the Rayleigh number is sufficiently large, the transition to a periodic flow around the fin on the sidewall of the cavity may happen and the oscillations induced by the fin in turn trigger traveling waves in the thermal boundary layer downstream of the fin.

The literature review shows that unsteady coupled thermal boundary layers around the partition with perturbations have not been investigated. However, the partition with a fin is widely present in industrial systems such as heat pipe exchangers and thus it is of practice significantly to investigate unsteady coupled thermal boundary layers induced by fins on the partition, which motivates this study.

In the remainder of this paper, the numerical procedures are presented in Section 2, the transition to unsteady coupled thermal boundary layers induced by the fin is described in Section 3, heat and mass transfer in the partitioned cavity is quantified in Section 4, and the major conclusions are summarized in Section 5.

## 2. Numerical procedures

The previous studies (e.g. [8,15]) show that two-dimensional (2D) numerical simulations may well describe transient natural convection flows in a differentially heated cavity. Accordingly, the 2D numerical procedure, similar to that in [24,25], was used to describe natural convection flows in a differentially heated partitioned cavity with a fin. The 2D governing equations may be written in the following non-dimensional forms,

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{1}$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{\partial p}{\partial x}+\frac{\Pr}{Ra^{1/2}}\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right), \tag{2}$$

$$\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=-\frac{\partial p}{\partial y}+\frac{\Pr}{Ra^{1/2}}\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)+\Pr T, \tag{3}$$

$$\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{1}{Ra^{1/2}}\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right). \tag{4}$$

Natural convection flows in the cavity are determined by three governing parameters: the Rayleigh number ($Ra$), the Prandtl number ($Pr$), and the aspect ratio ($A$) (also refer to [1]). They are defined as follows,

$$\mathrm{Ra}=\frac{g\beta(T_h-T_c)H^3}{\nu\kappa},\quad \Pr=\frac{\nu}{\kappa},\quad A=\frac{H}{L}. \tag{5}$$

Fig. 1 shows a schematic of the computational domain and boundary conditions. A vertical partition of zero-thickness vertically placed in the middle is perfectly conducting. The top and the bottom of the 2D computational domain and the horizontal fin with a length of 1/12 and a thickness of zero, placed on the mid-height of the partition, are assumed adiabatic. The sidewalls are regarded as isothermal, $T = -1/2$ on the left sidewall and $T = 1/2$ on the right sidewall. All wall boundaries of the computational domain are considered to be rigid and no-slip. Note that the coordinate origin is at the center of the cavity.

Fluent (Ansys 14.0) was used to solved the governing equations. The SIMPLE algorithm was used for the pressure velocity coupling. All second derivatives and linear first derivatives in Eqs. (2–4) were approximated by a second-order center-differenced scheme. The advection

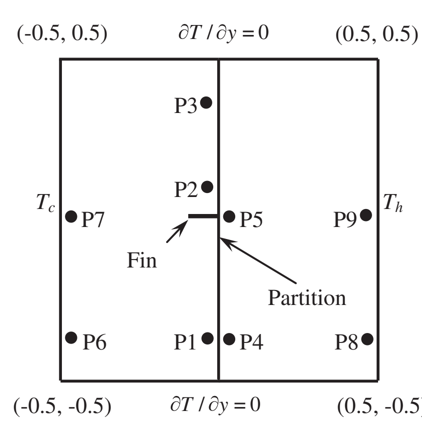


**Fig. 1.** Schematic of the computation domain and boundary conditions. P1 ($x = -0.0083$, $y = -0.375$), P2 ($x = -0.0083$, $y = 0.125$), P3 ($x = -0.0083$, $y = 0.375$), P4 ($x = 0.0083$, $y = -0.375$), P5 ($x = 0.0083$, $y = 0$), P6 ($x = -0.4927$, $y = -0.375$), P7 ($x = -0.4927$, $y = 0$), P8 ($x = 0.4927$, $y = -0.375$) and P9 ($x = 0.4927$, $y = -0.375$) record the points used in the subsequent figures.