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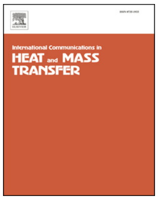

Analytical solution of thermally developing microtube heat transfer including axial conduction, viscous dissipation, and rarefaction effects ☆

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ABSTRACT

The solution of extended Gratez problem for micro-scale gas flows is performed by coupling of rarefaction, axial conduction and viscous dissipation at slip flow regime. The analytical coupling achieved by using Gram–Schmidt orthogonalization technique provides interrelated appearance of corresponding effects through the variation of non-dimensional numbers. The developing temperature field is determined by solving the energy equation locally together with the fully developed flow profile. Analytical solutions of local temperature distribution, and local and fully developed Nusselt number are obtained in terms of dimensionless parameters: Peclet number, Knudsen number, Brinkman number, and the parameter Kappa accounting temperature-jump. The results indicate that the Nusselt number decreases with increasing Knudsen number as a result of the increase of temperature jump at the wall. For low Peclet number values, temperature gradients and the resulting temperature jump at the pipe wall cause Knudsen number to develop higher effect on flow. Axial conduction should not be neglected for Peclet number values less than 100 for all cases without viscous dissipation, and for short pipes with viscous dissipation. The effect of viscous heating should be considered even for small Brinkman number values with large length over diameter ratios. For a fixed Kappa value, the deviation from continuum increases with increasing rarefaction, and Nusselt number values decrease with an increase in Knudsen number. Q4

1. Introduction

Interest in micro- and nanoscale heat transfer has been explosively increasing in accordance with the developments in MEMS and nanotechnology during the last two decades. The aim of cooling micro- and nanoscale devices is an important subject for most engineering applications. Cooling of devices having the dimensions of microns is a completely different problem than what is analyzed in the macro world which makes investigation of the flow characteristics of micro- and nanoscale flows a key research field.

One can understand some of the advantages of using micro- and nanoscale devices in heat transfer, starting from the single phase internal flow correlation for convective heat transfer,

$$h = \frac{Nu \cdot k}{D} \qquad (1)$$

where h is the convective heat transfer coefficient, Nu is the Nusselt number, k is the thermal conductivity of the fluid and D is the hydraulic diameter of the channel or duct. In internal fully developed laminar flows, Nu becomes a constant. Theory calculates $\mathrm{Nu} = 3.657$ for the constant wall temperature case, and $\mathrm{Nu} = 4.364$ for the constant heat flux case [1]. As Reynolds number (Re) is proportional to hydraulic diameter, fluid flow in channels of small hydraulic diameter will predominantly be laminar. The above correlation therefore indicates that the heat transfer coefficient increases as channel diameter decreases. As a result of the hydraulic diameter being order of tens or hundreds of micrometers in forced convection microscale applications, heat transfer coefficient should be extremely high. However, the question is whether the earlier mentioned theoretical Nu values are still the same for micro flows. While the system size is decreased to increase the surface to volume ratio and enhance the heat transfer, probable effects of micro-level small size onto transport characteristics should be carefully examined.

In a macroscale, continuum approach is the basis for most of the cases. However, continuum hypothesis may not be applicable for some of the micro-scale fluid transport and heat transfer problems, especially for micro gas flows. While the ratio of the average distance traveled by the molecules without colliding with each other, the mean free path (λ), to the characteristic length of the flow (L) is increases, the continuum approach fails to be valid, and the fluid modeling shifts from continuum model to molecular model. This ratio is known as Knudsen number ($\mathrm{Kn} = \lambda/L$) which is employed to determine the flow characteristics [2]. The flow is considered as continuum flow for small values of Kn (<0.01), and the well known Navier–Stokes equations together

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Nomenclature

Br	Brinkman number
C_1	coefficient in Eq. (11d)
c_p	constant pressure specific heat, J/kgK
D	tube diameter, m
F_m	tangential momentum accommodation coefficient
F_t	thermal accommodation coefficient
h	convective heat transfer coefficient, W/m^{2}K
Kn	Knudsen number, λ/L
k	thermal conductivity, W/mK
Nu	Nusselt number
Pe	Peclet number, $k/\rho C_p$
Pr	Prandtl number, ν/α
R	tube radius, m
r	radial coordinate
r^*	dimensionless radial coordinate
T	fluid temperature, K
u	velocity, m/s
u^*	dimensionless velocity
x	axial coordinate
x^*	dimensionless axial coordinate, $x/(R\,Pe)$
Greek symbols	
α	thermal diffusivity, m^2/s
γ	specific heat ratio
λ	mean free path, m
λ_n	eigenvalue
μ	dynamic viscosity, kg/ms
κ	coefficient in Eq. (11d)
ν	kinematic viscosity, m^2/s
Θ	dimensionless temperature, $(T - T_w)/(T_i - T_w)$
η	dimensionless radial coordinate, $\rho_s\,r/R$
ξ	dimensionless axial coordinate, $\rho_s^2(2 - \rho_s^2)\,x/(R\,Pe)$
ξ^*	dimensionless axial coordinate, $\rho_s^2(2 - \rho_s^2)\,x/(R)$
Subscripts	
i	inlet
s	slip
w	wall

Table 1 Q1

The first 20 eigenvalues and corresponding coefficients for Pe = 10, 5, and 2.

	Pe = 1		Pe = 5		Pe = 10	
n	λ_n	A_n	λ_n	A_n	λ_n	A_n
1	1.4298	1.6059	2.3853	1.5774	2.5969	1.5354
2	2.2776	−1.0736	4.5109	−1.0458	5.5469	−0.9861
3	2.8850	0.8589	5.9765	0.8447	7.7139	0.7795
4	3.3855	−0.7357	7.1579	−0.7341	9.4592	−0.6875
5	3.8211	0.6534	8.1744	0.6578	10.9532	0.6309
6	4.2119	−0.5935	9.0798	−0.6001	12.2780	−0.5865
7	4.5695	0.5475	9.9040	0.5546	13.4796	0.5487
8	4.9012	−0.5107	10.6653	−0.5177	14.5862	−0.5162
9	5.2117	0.4804	11.3763	0.4870	15.6171	0.4879
10	5.5048	−0.4549	12.0457	−0.4611	16.5856	−0.4633
11	5.7831	0.4331	12.6800	0.4388	17.5017	0.4418
12	6.0486	−0.4142	13.2842	−0.4194	18.3731	−0.4227
13	6.3029	0.3975	13.8621	0.4023	19.2055	0.4058
14	6.5474	−0.3827	14.4171	−0.3871	20.0039	−0.3906
15	6.7830	0.3694	14.9515	0.3735	20.7719	0.3770
16	7.0107	−0.3574	15.4675	−0.3612	21.5128	−0.3646
17	7.2313	0.3465	15.9669	0.3501	22.2292	0.3533
18	7.4454	−0.3366	16.4511	−0.3399	22.9235	−0.3430
19	7.6534	0.3274	16.9216	0.3305	23.5975	0.3336
20	7.8560	−0.3190	17.3793	−0.3219	24.2528	−0.3248

with the no-slip and no-temperature jump boundary condition are applicable for the flow field. For 0.01 < Kn < 0.1 flow is in slip-flow regime (slightly rarefied). For 0.1 < Kn < 10 flow is in transition regime (moderately rarefied). Finally, the flow is considered as free-molecular flow for large values of Kn (>10) (highly rarefied); the tool for dealing with this type flow is kinetic theory of gases, Direct Simulation of Monte Carlo (DSMC) [3] and Molecular Dynamics [4–8].

As the characteristic length of the system decreases, the effect of rarefaction comes into picture. Classical no-slip velocity and no-temperature jump boundary conditions are not valid for a rarefied fluid flow at micro/nanoscale. Since the fluid particles adjacent to the boundary surface are not in thermodynamic equilibrium with the wall, there would be slip velocity and temperature jump at the channel wall (For a more detailed discussion on these, the readers are referred to the textbook by Gad-el-Hak [9]). For the slip flow regime (0.01 < Kn < 0.1), slip-velocity and temperature-jump boundary conditions for a microtube can be defined as follows [2],

$$u_s = -\frac{2-\sigma_m}{\sigma_m}\lambda\left(\frac{du}{dr}\right)_{r=R} \tag{3}$$

$$T - T_s = -\frac{2-\sigma_t}{\sigma_t}\frac{2\gamma}{\gamma+1}\frac{\lambda}{Pr}\left(\frac{\partial T}{\partial r}\right)_{r=R}. \tag{4}$$

In these equations, σ_m is the tangential momentum accommodation coefficient, F_t is the thermal accommodation coefficient, and γ is the specific heat ratio. These slip flow models are successfully employed to consider the effect of rarefaction on microscale flow [10] while good agreements are obtained with experimental measurements [11].

Most of the existing studies on microscale heat transfer successfully used Eqs. (3) and (4) to consider non-continuum effects developed due to small scale rarefaction. However, additional complications occur at

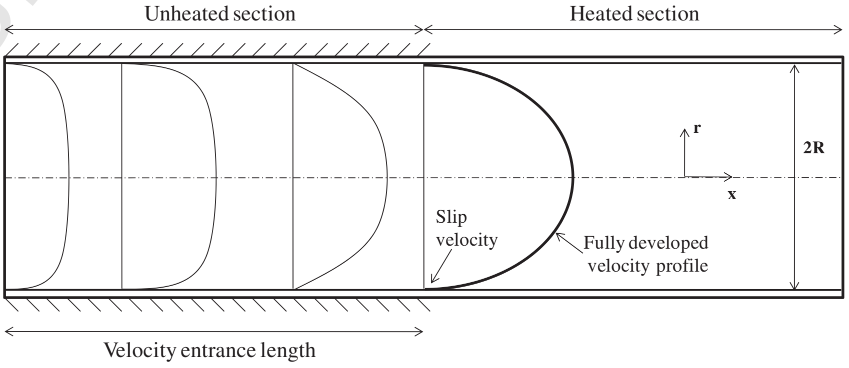

Fig. 1. Geometry of the problem.

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