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An inverse hyperbolic heat conduction problem in estimating pulse heat flux with a dual-phase-lag model ☆

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ABSTRACT

In this study, an inverse algorithm based on the conjugate gradient method and the discrepancy principle is applied to solve the inverse hyperbolic heat conduction problem with the dual-phase-lag heat transfer model in estimating the unknown boundary pulse heat flux in an infinitely long solid cylinder from the temperature measurements taken within the medium. An efficient numerical scheme involving the hybrid application of the Laplace transform and control volume methods in conjunction with hyperbolic shape functions is used to solve the hyperbolic direct problem. The inverse solutions will be justified based on the numerical experiments in which two different heat flux distributions are to be determined. The temperature data obtained from the direct problem are used to simulate the temperature measurements. The influence of measurement errors upon the precision of the estimated results is also investigated. Results show that an excellent estimation on the time-dependent pulse heat flux can be obtained for the test cases considered in this study.

1. Introduction

The advancement of micro- or nano-electromechanical systems (MEMS/NEMS) has made the various devices perform the advanced functions with a more compact size. At the same time, due to the development of short-pulse laser technologies, laser micro-machining, laser pattering, and laser hardening have been applied in the processes of making such electromechanical devices [1,2]. In the applications of metal processing, short-pulse laser heating of metals involves deposition of radiation energy on electrons, resulting in energy increase in electrons; energy is transferred to lattice through electron–lattice interaction and propagates through media [3].

In treating short-pulse laser heating problems, two different kinds of models have been usually employed in the literatures. One is the macroscopic thermal wave model, which was postulated by Vernotte [4] and Cattaneo [5], leading to a hyperbolic heat conduction equation and suggesting a finite speed propagation of heat in the medium. The other includes those from microscopic point of view, the two-step model [6] and the pure phonon field model [7], which were developed for evaluating the thermal transport phenomena in metal and dielectric solid, respectively. The two models suggest the microscale behavior of heat conduction, which neither the macroscopic thermal wave model nor the Fourier thermal diffusion model can do. To fill the gap from micro- to macroscopic theories, another form of heat conduction model, namely the Jeffreys type heat-flux equation, was given by Joseph and Preziosi [8] in 1989 by applying some ideas for describing shear waves in liquids. In this equation, two time constants are introduced, which are regarded as the relaxation time and the retardation time of the heat flow. Several years later, Tzou [9] deduced the same type of equation by adding two time constants, namely the phase-lags of temperature gradient and heat flux, in the Fourier heat flux equation. This model, called dual-phase-lag (DPL) model, lumps the micro-structural effects into the delayed response in time.

To consider the effect of micro-structural interactions on the fast-transient process of heat transport, the DPL model accounts for both the temporal and the spatial effects of heat transfer in one-temperature formulation and takes the form [9,10]:

$$q\left(r, t+\tau_q\right) = -k\nabla T(r, t+\tau_T), \tag{1}$$

where T is the temperature, k is the heat conductivity, q is the heat flux, t is the time, and r is the space variable. τ_q is the phase lag of the heat flux and τ_T is the phase lag of the temperature gradient. The heat flux precedes the temperature gradient for $\tau_q < \tau_T$. On the other hand, the temperature gradient precedes the heat flux for $\tau_q > \tau_T$. With the DPL model, not only can the temperature gradient precede the heat flux, the heat flux may precede the temperature gradient.

In inverse analyses, the direct heat conduction problems are concerned with the determination of temperature at interior points of a region when the initial and boundary conditions, heat generation, and material properties are specified, whereas, the inverse heat conduction problem (IHCP) involves the determination of the surface conditions, energy generation, thermophysical properties, etc., from the knowledge

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Nomenclature

c	specific heat (J kg^{-1} K^{-1})
f	dimensionless surface heat flux defined in Eq. (8)
J	functional
J'	gradient of functional
k	thermal conductivity (W m^{-1} K^{-1})
l	dimensionless distance between two neighboring nodes
p	direction of descent
q	surface heat flux (W m^{-2})
q_0	reference surface heat flux (W m^{-2})
r	space coordinate in r-direction (m)
r_o	radius of the cylinder (m)
s	Laplace transform parameter
T	temperature (K)
T_0	initial temperature (K)
T_r	reference temperature $r_o q_0/k$ (K)
t	time (s)
r_m	temperature measurement position (m)

Greek symbols

Δ	small variation quality
Λ	parameter defined in Eq. (8)
α	thermal diffusivity, $k/\rho c$ (m^2 s^{-1})
β	step size
γ	conjugate coefficient
ε	very small value
ς	transformed dimensionless time, $\xi_f - \xi$
η	dimensionless space coordinate in r-direction
η_m	dimensionless temperature measurement position, r_m/r_o
θ	dimensionless temperature
λ	variable used in the adjoint problem
μ	parameter defined in Eq. (16)
ξ	dimensionless time
ρ	density (kg m^{-3})
σ	standard deviation
τ_q	phase lag of the heat flux (s)
τ_T	phase lag of the temperature gradient (s)
τ_{Tq}	thermal lag ratio, τ_T/τ_q
ψ	parameter defined in Eq. (17)
ϖ	random variable

Superscript/subscript

N	iterative number

of temperature measurements taken within the body. To date, a variety of analytical and numerical techniques have been developed for the solution of the inverse heat conduction problems, for example, the conjugate gradient method (CGM) [11–14], the Tikhonov regularization method [15], the genetic algorithm [16], and the linear least-squares error method [17], etc.

The direct heat conduction problems based on the dual-phase-lag model has been studied by many authors. For example, based on a non-equilibrium heat transfer model, Zhang [18] obtained the bio-heat equations in the living tissue from the nonequilibrium model. Ramadan and Al-Nimr [19] numerically studied the thermal wave transmission and reflection phenomena induced by a pulsed boundary heat flux in a two-layer slab with imperfect interface. Ordóñez-Miranda and Alvarado-Gil [20] investigated the one-dimensional thermal wave transport in a semi-infinite medium and obtained the formulas to determine the difference of the time delays as well as other thermal properties of the semi-infinite layer. Recently, Lee et al. [21] interpreted the DPL thermal behavior in a thin metal film exposed to short-pulse laser heating. A hybrid numerical scheme is used to solve the hyperbolic heat conduction equation.

However, to the best of authors' knowledge, the study on the inverse heat conduction problems based on the DPL model is still limited in the literature. For example, Huang and Lin [22] applied the CGM with the dual-phase-lag model to estimate the unknown heat generation, due to the ultra-short duration laser heating, based on the interior temperature measurements. Liu and Lin [23] applied a hybrid method of the Laplace transform, change of variables, and the least-squares scheme to estimate the phase lag times of a bi-layered spherical tissue based on the DPL model from the experimental data. Recently, Lee et al. [24] solved the inverse hyperbolic heat conduction problem with a DPL model to estimate the unknown space- and time-dependent energy absorption rate in a thin metal film exposed to short-pulse laser heating.

The focus of the present study is to develop an inverse hyperbolic heat conduction analysis based on the DPL model for estimating the unknown pulse heat flux at the boundary of an infinitely long solid cylinder, from the knowledge of temperature measurements taken within the medium. An analysis of this kind poses significant implications on several applications of metal processing, such as laser micromachining, laser pattering, and laser hardening. Here, we present the conjugate gradient method and the discrepancy principle [25] to estimate the unknown boundary pulse heat flux by using the simulated temperature measurements. The CGM derives from the perturbation principles and transforms the inverse problem to the solution of three problems, namely, the direct, sensitivity and the adjoint problem, which will be discussed in detail in the following sections.

2. Direct problem

To illustrate the methodology for developing expressions to use in determining unknown pulse heat flux in an inverse hyperbolic heat conduction problem with a DPL model, an infinitely long solid cylinder with radius r_o and constant thermal properties is considered. The linearized DPL model of heat transfer in the cylinder is [26]:

$$\tau_q \frac{\partial q(r,t)}{\partial t} + q(r,t) = -k\frac{\partial T(r,t)}{\partial r} - k\tau_T \frac{\partial^2 T(r,t)}{\partial t \partial r}, \text{ in } 0 \le r \le r_o, t > 0, \quad (2)$$

where k, q, τ_q, and τ_T are the thermal conductivity, heat flux, phase lag for heat flux (relaxation time), and phase lag for temperature gradient, respectively.

In a local energy balance, the one-dimensional energy equation without internal energy source is given as:

$$\frac{\partial q(r,t)}{\partial r} + \frac{1}{r} q(r,t) = -\rho c \frac{\partial T(r,t)}{\partial t}, \quad (3)$$

where ρ is the density and c is the specific heat. Substituting Eq. (2) into Eq. (3) leads to the DPL heat conduction equation as:

$$\frac{\alpha}{r}\frac{\partial}{\partial r}\left[r\left(\frac{\partial T}{\partial r} + \tau_T \frac{\partial}{\partial t}\frac{\partial T}{\partial r}\right)\right] = \tau_q \frac{\partial^2 T}{\partial t^2} + \frac{\partial T}{\partial t}, \quad \text{in } 0 \le r \le r_o, \quad t > 0, \quad (4)$$

where $\alpha = k/\rho c$ is the thermal diffusivity.

Initially, the solid cylinder is at a uniform temperature T_0, while the heat flux density distribution is zero. For $t > 0$, the outer boundary surface is subjected to a pulse heat flux $q(t)$. Then, the boundary and initial conditions for the problem are as follows:

$$\frac{\partial T(r,t)}{\partial r}, \text{at } r = 0, \quad (5)$$