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# Multi-objective optimization of natural convection in a cylindrical annulus mold under magnetic field using particle swarm algorithm☆

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ABSTRACT

In the continuous casting process, natural convection occurs in mold containing a liquid metal. Natural convection in the melt causes the impurities to move and this phenomenon can lead to poor product. Therefore, by reducing natural convection, the quality of the product is improved. In this paper, 3D numerical simulation and multi-objective optimization of natural convection in a cylindrical annulus mold filled with molten potassium under a magnetic field is carried out. The inner and outer cylinders are maintained at uniform temperatures and other walls are thermally insulated. Two objective functions including the natural convection heat transfer rate (average Nusselt number) and magnetic field strength have been considered simultaneously. The multi-objective particle swarm optimization algorithm (MOPSO) has been employed. Four decision variables are the Hartmann number, inclination angle, and magnetic field angles. For the optimization process, the calculations of three-dimensional Navier–Stokes, energy, and electrical potential equations are combined with MOPSO. Using the numerically evaluated objective functions, the optimum frontier is estimated by a second order polynomial based on objective functions.



## 1. Introduction

In the continuous casting process, natural convection occurs in an enclosure filled with a molten metal due to a temperature difference between the solid walls and the molten metal. Natural convection flow in the melt causes the impurities to move. This phenomenon affects the structure of the final product, including ingot, metal slab, and the like. Hence, the melt must have a uniform temperature throughout the enclosure to reduce thermal stresses and natural convection. In recent decades, this problem has been solved by using Magnetohydrodynamics, and natural convection flow has decreased within the enclosures by applying a magnetic field. In the Magnetohydrodynamics, electrically conducting fluid flow in the presence of a magnetic field is investigated. When molten metal (electrically conducting fluid) motion is subjected to a magnetic field, the Lorentz force is generated. It can control growing crystals. This force affects the buoyancy force; thus, the resulting natural convection is suppressed. In these processes, the study and understanding of heat transfer are important, owing to the better control in the process of producing high-quality products. Consequently, the rate of heat transfer convection should be minimized in this process. Gao et al. [1] proved that imposing a magnetic field leads to products of better quality. They demonstrated that totally equiaxed grains could be obtained in pure aluminum under the external effect of the pulsed magnetic field; yet, only thin columnar grains are formed in high-melting steel, even when treated by the higher magnetic intensity. An analysis of the hydromagnetic natural convection is generally employed in many industrial applications including metal solidification, cooling of electronic equipment, solar technology, heat exchangers, and the like [2–6]. Natural convection in a rectangular cavity has been the subject of a vast number of investigations [7–10]. The annular geometry has been widely investigated by many researchers using experimental, analytical, and numerical methods. Numerically, Afrand et al. [11] studied a steady, laminar, and natural-convection flow under different directions of uniform magnetic field in a long horizontal annulus containing gallium with isothermal walls. Their results revealed that when the strength of a magnetic field is increased, the convection heat transfer is decreased. In another research, they carried out three-dimensional numerical investigations of natural convection in a tilted cylindrical annulus containing molten potassium under various magnetic fields. Their computational results confirmed the effect of the magnetic field direction on natural convection [12]. Wrobel et al. [13] studied convection

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**Nomenclature**

| | |
|---|---|
| $B_0$ | magnitude of the external magnetic field ($kg/s^2A$) |
| $D$ | annulus gap; $D = r_o - r_i(m)$ |
| $E$ | dimensional induced electric field ($mkg/s^3A$) |
| $E^*$ | dimensionless induced electric field |
| $F$ | Lorentz force ($N/m^3$) |
| $g$ | acceleration due to gravity ($m/s^2$) |
| $Ha$ | Hartmann number |
| $J$ | electric current density ($A/m^2$) |
| $L$ | height of the annulus |
| $Nu$ | Nusselt number |
| $P$ | pressure ($N/m^2$) |
| Pr | Prandtl number |
| $Ra$ | Rayleigh number |
| $T$ | dimensional temperature ($K$) |
| $T^*$ | dimensionless temperature |
| $(r, z)$ | radial and axial co-ordinates |
| $(R, Z)$ | dimensionless radial and axial co-ordinates |
| $(r_i, r_o)$ | radii of inner and outer cylinders ($m$) |
| $(u, v, w)$ | dimensional velocity components in $(r, \theta, z)$ direction ($m/s$) |
| $(U, V, W)$ | dimensionless velocity components in $(r, \theta, z)$ direction |
| $(x, y, z)$ | Cartesian c-ordinate components |

*Greek letters*

| | |
|---|---|
| $\alpha$ | thermal diffusivity ($m^2/s$) |
| $\beta$ | fluid coefficient of thermal expansion ($1/K$) |
| $\delta, \eta$ | magnetic field angles |
| $\varphi$ | dimensional electrical potential ($m^2kg/s^3A$) |
| $\Phi$ | dimensionless electrical potential |
| $\gamma$ | inclination angle |
| $\lambda$ | radii ratio |
| $\mu$ | dynamic viscosity ($kg/ms$) |
| $\theta$ | azimuthal angle |
| $\rho$ | fluid density ($kg/m^3$) |
| $\sigma$ | fluid electrical conductivity ($s^3A^2/m^3kg$) |

*Subscripts*

| | |
|---|---|
| $h$ | condition at hot wall |
| $c$ | condition at cold wall |

in an annular enclosure with a round rod core and a cylindrical outer wall filled with paramagnetic fluid using an experimental and numerical analysis. It is concluded that the magnetic field yielded heat transfer values four times higher than that of the thermal Rayleigh number; thus, it influences heat transfer more efficiently in comparison with increasing the thermal Rayleigh number. Steady, fully developed laminar natural convective flow in open-ended vertical concentric annuli in the presence of a radial magnetic field was considered analytically by Singh and Singh [14]. They corroborated that the Hartmann number and the gap between cylinders play an important role in controlling the behavior of fluid flow. Natural convection in concentric horizontal annuli containing magnetic fluid under non-uniform magnetic fields was investigated experimentally by Sawada et al. [15]. Two concentric cylinders were made of copper and placed horizontally and were maintained at constant temperatures. Various kinds of experiments were performed to clarify the effects of the direction and strength of the magnetic fields on the natural convection. Sankar et al. [16] used an implicit finite difference scheme to simulate the effect of the axial or radial magnetic field on the natural convection in a vertical cylindrical annulus at a low Prandtl number. It was found that the flow and heat transfer were suppressed more effectively by an axial magnetic field in a shallow cavity, whereas in tall cavities, a radial magnetic field was more effective. Furthermore, it was observed that the average Nusselt number increased with the radii ratio but decreased with the Hartmann number. Kakarantzas et al. [17] investigated laminar and turbulent regimes of the liquid metal flow in a vertical annulus under constant horizontal magnetic field numerically. Their results illustrated that when the magnetic field increases, the flow becomes laminar.

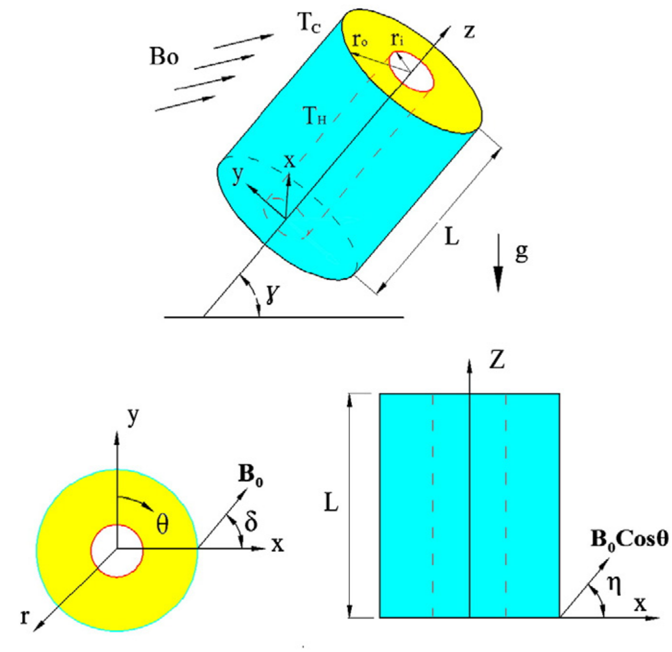

**Fig. 1.** Geometry and coordinates of cavity configuration with magnetic effect.

Optimization is applied in many fields such as engineering and economic in order to reduce the cost and enhance the quality. In this regard, in recent years, many researches have performed on heat transfer optimization in many different ways [18–20]. In addition, convective heat transfer optimization has been studied by many researchers including Burger et al. [21], Liu et al. [22], Mehrabi et al. [23], Lee and Kim [24], Chen et al. [25], and Jia et al. [26].

By reviewing previous researches, it can be found that most of them have focused on horizontal or vertical cylindrical annulus under constant direction magnetic field. Also, these researches indicated that for an enclosure with constant size and thermo-physical properties, the four parameters including the strength of the magnetic field, the inclination angle of the enclosure, and the spatial angles of the magnetic field are effective on the heat transfer rate [11,12]. It is interesting that despite the attractiveness of optimization problems for engineers, research on multi-objective optimization for convective heat transfer and as the objective functions is not performed. The purpose of this study is to minimize the convective heat transfer by applying a optimized magnetic field in a cylindrical annulus mold containing molten potassium. To achieve this goal, the multi-objective particle swarm optimization algorithm is combined with the SIMPLER algorithm using a FORTRAN computer code.

## 2. Mathematical formulation

The modeling considered in the present study is an inclined cylindrical annulus formed by two concentric cylinders of inner and outer radii, $r_i$ and $r_o$, respectively, as shown in Fig. 1. The inner and outer cylinders are maintained at isothermal but different temperatures, $T_H$ and $T_C$, respectively. The top and bottom walls are assumed to be adiabatic.