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# Heat transfer and entropy generation of turbulent forced convection flow of nanofluids in a heated pipe☆

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ABSTRACT

Eulerian–Eulerian multi-phase mixture model is applied to numerically analyse the turbulent flow and heat transfer behaviour of water based $Al_2O_3$ and $TiO_2$ nanofluids in a pipe. The main goal of the present work is to investigate the effects of volume concentrations, Brownian motion and size diameter of nanoparticles on the flow and heat transfer. Analysis of entropy generation is presented in order to investigate the condition that optimises the thermal system. Results reveal that small diameter of nanoparticles with their Brownian motion has the highest heat transfer rate as well as thermal performance factor for $\chi = 6\%$. Above all, the higher heat transfer rate is found while using the multi-phase model than the single-phase model (Saha and Paul [1]). Also, the optimal Reynolds number is found to be $Re = 60 \times 10^3$ for $\chi = 6\%$ and $d_p = 10$ nm, which minimises the total entropy generation. Finally, it is showed that $TiO_2$–water nanofluid is the most energy efficient coolant than $Al_2O_3$–water nanofluid, and some new correlations have been proposed for the calculation of average Nusselt number.



## 1. Introduction

Forced convection heat transfer is an important phenomenon in many engineering applications e.g. cooling of electronic components, nuclear energy, solar energy, transportation, building heating, lubrication technologies and so on. However, low thermal conductivity of conventional fluids such as water, air, engine oil and ethylene glycol is the primary limitation on the enhancement of heat transfer performance in such engineering applications. In order to achieve a better performance on the heat transfer, highly conductive nano particles are suspended in base fluid to form a nanofluid and various applications of nanofluids are highlighted in Saha and Paul [1]. Nanofluid becomes a promising alternative approach for engineering and thermal applications and research is underway to apply nanofluids in applications where the conventional heat transfer fluids are not capable of improving the desired heat transfer rate. Relevant research that covers the area of applications of turbulent nanofluid flows and heat transfer in circular pipes with single–phase assumption has been discussed in Saha and Paul [1]. We therefore particularly focus our attention here to the experimental and numerical studies which have been conducted in turbulent nanofluid flow with multi-phase approach.

For the first time, Behzadmehr et al. [2] numerically examined the turbulent forced convection heat transfer in a circular tube using Cu–water nanofluid with a two-phase mixture model. Their investigations show that the multi-phase model is more accurate than the single phase model. Maiga et al. [3] studied numerically the turbulent flow and heat transfer behavior of $Al_2O_3$–water nanofluid at various nanoparticle volume concentrations in a circular tube. In this study, $Re = 10^4$ to $5 \times 10^5$ and the fluid inlet temperature of 293.15 K were considered. Also, the effect of nanoparticle volume fraction and Reynolds number was presented and a new correlation was proposed. Their numerical outcomes revealed that the inclusion of nanoparticles into the base fluid enhanced the heat transfer rate with the increase of nanoparticle volume fraction. Similar investigation was carried out by Bianco et al. [4] using both single-phase and multi-phase approaches and it was also found that the accuracy of the multi-phase mixture model is better than the single-phase model.

Namburu et al. [5] analysed numerically the forced convective flow and heat transfer behaviour of EG-water based CuO, $Al_2O_3$ and $SiO_2$ nanofluids flow through a circular tube. It is shown that nanofluids have higher viscosity, thermal conductivity and heat transfer rate compared to the base fluid. Akbari et al. [6] carried out a numerical investigation on the turbulent forced convection flow in a horizontal tube. It is observed that the thermal predictions by two-phase model are very sensitive to the particle volume concentration, and the single-phase and two-phase models predict almost identical hydrodynamic fields. Kumar [7] studied numerically the heat transfer behaviour of $Al_2O_3$–water nanofluid using the single phase approach covering both laminar and turbulent flow regime. In this study, $Re = 10 \times 10^3$ to $30 \times 10^3$ and the fluid inlet temperature 315 K are considered. It is found that heat transfer rate significantly enhanced in the turbulent flow regime compared to that in the laminar flow regime.

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**Nomenclature**

| | |
|---|---|
| $a$ | Acceleration ($m/s^2$) |
| $A_0, A_1, C_1, C_2, C_\mu$ | Model constant |
| $C_p$ | Specific heat capacity (J/kg K) |
| $D$ | Einstein diffusion coefficient |
| $D_h$ | Diameter of a pipe (m) |
| $d_f$ | Fluid molecular diameter (m) |
| $d_p$ | Diameter of nanoparticle (nm) |
| $E_{gen}$ | Entropy generation (W/K) |
| $f$ | Darcy friction factor |
| $f_{drag}$ | Drag function |
| $f_\mu$ | Dumping function |
| $G_k$ | Generation of turbulent kinetic energy |
| $H$ | Enthalpy (J/kg) |
| $I$ | Turbulent intensity |
| $L$ | Length (m) |
| $M$ | Molecular weight of the base fluid |
| $\dot{m}$ | Mass flow rate (kg/s) |
| $N$ | Avogadro number |
| $N_x, N_r$ | Number of grid distribution in axial and radial directions |
| $Nu$ | Nusselt number |
| $P$ | Pressure ($N/m^2$) |
| $Pr$ | Prandtl number |
| $\dot{q}_s$ | Heat flux of the pipe ($W/m^2$) |
| $R$ | Radius of a pipe (m) |
| $Re$ | Reynolds number |
| $r$ | Radial coordinate (m) |
| $S$ | Modulus of the mean rate of strain tensor |
| $T, t$ | Time average and fluctuating temperature (K) |
| $u_B$ | Nanoparticle particle mean Brownian velocity (m/s) |
| $u_\tau$ | Friction velocity (m/s) |
| V, $v$ | Time average and fluctuating velocity components (m/s) |
| $x$ | Axial coordinate (m) |

*Greek symbols*

| | |
|---|---|
| $\rho$ | Density ($kg/m^3$) |
| $\mu$ | Dynamic viscosity (kg/ms) |
| $\lambda$ | Thermal conductivity (W/m K) |
| $\kappa$ | Turbulent kinetic energy ($m^2/s^2$) |
| $\epsilon$ | Dissipation rate of turbulent kinetic energy ($m^2/s^3$) |
| $\nu$ | Kinematic viscosity ($m^2/s$) |
| $\sigma_t$ | Constant of turbulent Prandtl number |
| $\mu_t$ | Turbulent molecular viscosity |
| $\sigma_\kappa$ | Effective Prandtl number for turbulent kinetic |
| $\sigma_\epsilon$ | Effective Prandtl number for rate of dissipation |
| $\chi$ | Nanoparticle volume concentration |
| $\xi$ | Thermal performance factor |
| $\tau_D$ | Time (s) |
| $\overline{\tau_\tau}$ | Ratio of average shear stresses |

*Subscripts*

| | |
|---|---|
| avg | Average |
| eff | Effective |
| f | Base fluid |
| fl | Frictional |
| fr | Freezing |
| in | Inlet |
| m | Mixture |
| mean | Mean |
| nf | Nanofluid |
| out | Outlet |
| p | Nanoparticle |
| s | Secondary phase |
| w | Wall |
| t | Turbulent |
| Th | Thermal |

Most recently, Saha and Paul [1] have examined the effect of volume concentration, diameter and Brownian motion of nanoparticles on the convective heat transfer of $Al_2O_3$ and $TiO_2$–water nanofluids using a single-phase numerical model. The aim of this piece of work is to extend the numerical model to investigate the effects of multi-phase flow of $Al_2O_3$ and $TiO_2$–water nanofluids. Particular attention is paid to the entropy generation of these two nanofluids and importantly, how the performance factor of nanofluids is varied if the numerical simulation is switched from the single- to multi-phase model.

## 2. Mathematical modelling

Eulerian–Eulerian mixture model is used to model the multi-phase flows with the assumption that the phases between fluid and solid particles move at a same velocity with a very strong coupling between them. Also, the phases are supposed to be interpenetrating, that means each phase has its own velocity vector field and within any control volume there is a volume concentration of each phase. It should also be noted that the mixture model solves the continuity, momentum and energy equations for the mixture and the volume fraction equation for the secondary phases. An axi-symmetric model is considered to describe the characteristics of nanofluids flowing through a straight circular pipe under a constant heat flux boundary condition and within a turbulent flow regime. It consists of a pipe with length $L$ of 1.0 m and a circular section with diameter, $D_h$ of 0.019 m as shown in Fig. 1. The flow and thermal fields are assumed to be axisymmetric with respect to the horizontal plane parallel to the $x$-axis.

## 3. Governing equations

The dimensional steady-state governing equations of fluid flow and heat transfer for the two-phase mixture model have been presented and the following assumptions are made:

i. Fluid flow is incompressible, Newtonian and turbulent.
ii. The Boussinesq approximation is negligible as the pipe is placed horizontally.
iii. Nanoparticles are spherical and uniform in size and shape.
iv. The compression work and the viscous dissipation are negligible.

Under the above assumptions, the governing equations for the mixture model can be expressed as [8]:

Continuity equation:

$$\nabla.\left(\rho_m \vec{V}_m\right)=0 \tag{1}$$

Momentum equation:

$$\nabla.\left(\rho_m \vec{V}_m \vec{V}_m\right)=-\nabla P_m+\nabla.\left[\mu_m \nabla \vec{V}_m-\sum_{s=1}^{n}\chi_s\rho_s\overline{v}_s\overline{v}_s\right]+\nabla.\left(\sum_{s=1}^{n}\chi_s\rho_s\vec{V}_{dr,s}\vec{V}_{dr,s}\right) \tag{2}$$

Energy equation:

$$\nabla.\left[\sum_{s=1}^{n}\chi_s\vec{V}_s(\rho_s H_s+P_m)\right]=\nabla.\left(\lambda_m\nabla T-C_p\,\rho_m\overline{vt}\right) \tag{3}$$