International Communications in Heat and Mass Transfer 61 (2015) 88–95

Contents lists available at ScienceDirect

International Communications in Heat and Mass Transfer

journal homepage: www.elsevier.com/locate/ichmt

Solution of Radiative Transfer Equation using Discrete Transfer Method for two-dimensional participating medium ☆

Himanshu Nirgudkar, Sumit Kumar, Atul Srivastava *

Department of Mechanical Engineering, Indian Institute of Technology Bombay, Powai, Mumbai 400076, India

ARTICLE INFO

Available online 27 December 2014

Keywords:
Radiative Transfer Equation (RTE)
Discrete Transfer Method (DTM)
Two-dimensional participating medium
Laser-irradiated biological phantoms

ABSTRACT

The present work reports the development and application of a simplified numerical approach for solving the transient Radiative Transfer Equation (RTE) using Discrete Transfer Method (DTM) in two-dimensional coordinate system. The numerical formulation of the proposed scheme is discussed in detail and its application in the context of understanding light propagation phenomenon in laser-irradiated numerically simulated biological tissue phantoms has been demonstrated. The developed mathematical model has first been benchmarked against the results published in the literature for the same operating conditions. Thereafter, the results of a detailed parametric study have been presented to investigate the effects of optical properties of the biological phantom on the intensity distribution within the two-dimensional tissue phantom, net transmittance and reflectance, etc. The effect of anisotropy of the tissue medium has also been studied to understand the phenomenon of light propagation within the body of the sample. Based on the results of the study, it has been inferred that the developed numerical methodology for two-dimensional Discrete Transfer Method successfully predicts the physics of the phenomena of light propagation within the tissue phantom and compares well with the other conventionally employed numerical models for solving the Radiative Transfer Equation.

1. Introduction

Radiative heat transfer has been an area of active research in the recent past. Traditionally, the radiative heat transfer within a participating medium is modelled using the Radiative Transfer Equation (RTE) proposed by Chandrasekhar [1] and since then it has been applied to various areas such as studying heat transfer phenomena in furnaces, boilers, and internal combustion engines and in applications like photo-thermal therapy, optical tomography and pulsed laser interaction with materials, etc. [2,3]. The RTE is a complex integro-differential equation which can be solved numerically to obtain the intensity and subsequently the heat flux distribution within the medium. Various schemes such as the zonal method, 2-flux method, Monte-Carlo method, Discrete Transfer Method (DTM), Discrete Ordinate Method (DOM), and Finite Volume Method (FVM) have been used to obtain the solution of the Radiative Transfer Equation [4]. However, the zonal method has its limitation in handling complicated geometries and cannot be coupled with the flow-field and energy equations. The Monte-Carlo method can handle complex geometries but its coupling with the energy equation for determination of temperature field is difficult. Moreover, the computational time required is extremely high. DTM is a hybrid of the zonal method and Monte-Carlo method [2,5–10]. In this method, the energy emitted is divided into the hemisphere along finite number of rays, the radiation leaving the surface element in a certain range of solid angles can be approximated by the single ray [11].

DTM was first proposed by Lockwood and Shah [5] for studying steady state radiation heat transfer in combustors. The numerical solution of transient RTE using DTM was later discussed by various authors [2,3]. A comparative study of DOM, DTM and FVM has been presented by Mishra et al. [3] for a one-dimensional medium subjected to short pulse laser irradiation. The approximation error in determining heat flux using DTM has been reported by Versteeg et al. [12]. Coelho and Carvalho [13] examined that the original formulation of DTM used is generally non-conservative i.e. it does not satisfy the principle of conservation of energy.

In recent times, DTM has attracted the attention of various researchers as the method offers an advantage in terms of its applicability for complex geometries as compared to DOM. Coelho [14] has compared the accuracy of results obtained using DTM and DOM for radiative heat transfer in non-grey gases. It was reported that for such applications, the DTM predicts more accurate results and fares better in comparison with DOM. Also, DTM, being a ray-tracing method, can be applied to solve RTE in a medium with varying refractive index, as reported by Krishna and Mishra [7]. The trajectory of light intensity propagation can be determined using Snell's law, along which the RTE can be solved. However, these studies are limited to one-dimensional domain only and a detailed methodology for solving the transient form of RTE for two-dimensional medium using DTM is not available in the

☆ Communicated by W.J. Minkowycz.

* Corresponding author.
E-mail addresses: atulsr@iitb.ac.in, atuldotcom@gmail.com (A. Srivastava).

http://dx.doi.org/10.1016/j.icheatmasstransfer.2014.12.003
0735-1933/

Nomenclature

a	anisotropy factor
c	velocity of light in medium
$\hat{n}$	unit vector normal to the surface
p	scattering phase function
q	heat flux
s	distance travelled by beam
$\hat{s}$	unit vector in s direction
t	time
t_p	pulse width of laser
M	number of divisions
G	irradiation
I	intensity
S	source

Greek symbols

σ_s	Stefan–Boltzmann's constant
Ω	solid angle
β	extinction coefficient
θ, ϕ	polar and azimuthal angle
κ	absorption coefficient
μ, ζ	Direction cosines in x and y directions
σ	scattering coefficient
τ	optical thickness
ω	scattering albedo
ϵ	emissivity

Subscripts

av	average
c	collimated
d	diffused
w	wall

Superscripts

m	index for a discrete direction
D	Downstream
U	Upstream
$*$	non-dimensional parameter

literature. With this background, the present work reports a simplified methodology to solve the transient form of RTE in two-dimensional participating medium using DTM.

2. Mathematical formulation

Consider a two dimensional medium subjected to short pulse laser irradiation at its top surface. As shown in Fig. 1, the 2-D domain has been discretized into a number of control volumes. Consider a point P_1 on the boundary control volume at the centre of one of the cell faces. We "fire" a number of rays (M_θ) in predefined directions (θ_l) till it intersects the other boundary surfaces at points Q_j. In Fig. 1, P_1Q_1 is the ray from the boundary point P_1 at an angle Ω_{P1Q1}. The intensity along the ray P_1Q_1 is represented as I_{P1Q1}. Similarly ray P_1Q_2 is at an angle Ω_{P1Q2}. From a particular point Q_j, the ray is then traced backwards up to the point P and the discretized Radiative Transfer Equation is solved along that particular ray for each control volume. The point at which a particular ray enters a given control volume is termed as the Upstream point (U) and the point where it leaves the control volume is denoted by the Downstream point (D). The intensity at the downstream point is then determined knowing the intensity value at the upstream point (U).

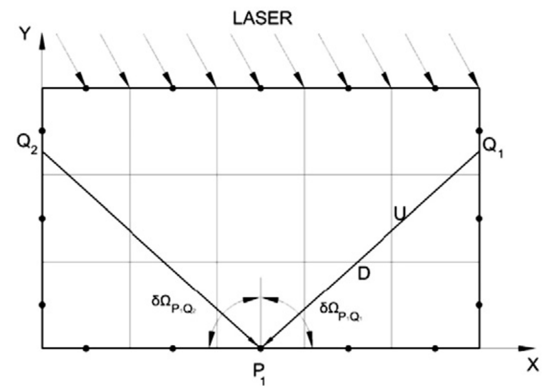

Fig. 1. Discretization of domain.

2.1. Radiative Transfer Equation

The RTE describes the propagation of light of intensity I within a participating medium. The transient form of RTE in terms of extinction coefficient ($=\kappa+\sigma$) can be expressed as [14,15]

$$\frac{1}{c}\frac{\partial I}{\partial t}+\frac{\partial I}{\partial s}=-\beta I+\kappa I_b+\frac{\sigma}{4\pi}\int_{4\pi} I\left(\hat{s}'\right)p\left(\hat{s}',\hat{s}\right)d\Omega'. \tag{1}$$

The intensity I within the medium is composed of two parts, namely the collimated intensity I_c and diffused intensity I_d [3].

$$I=I_c+I_d. \tag{2}$$

The variation of collimated component within the medium is given by the Beer–Lambert's law as

$$\frac{\partial I_c}{\partial s}=-\beta I_c. \tag{3}$$

Substituting Eqs. (2) and (3) in Eq. (1) yields

$$\frac{1}{c}\frac{\partial I_c}{\partial t}+\frac{1}{c}\frac{\partial I_d}{\partial t}+\frac{\partial I_d}{\partial s}=-\beta I_d+S_c+S_d \tag{4}$$

$$\text{where } S_c=\frac{\sigma}{4\pi}\int_{4\pi} I_c\left(\hat{s}'\right)p\left(\hat{s}',\hat{s}\right)d\Omega' \text{ and } S_d=\frac{\sigma}{4\pi}\int_{4\pi} I_d\left(\hat{s}'\right)p\left(\hat{s}',\hat{s}\right)d\Omega'+\kappa I_b. \tag{5}$$

S_c and S_d are respectively the collimated source term and diffuse source term.

In order to evaluate the source term, for linear isotropic phase function, we have

$$p\left(\hat{s}',\hat{s}\right)=\left(1+a\,\hat{s}\cdot\hat{s}'\right). \tag{6}$$

Here a represents the anisotropic factor.

In terms of non-dimensional time $t^*=\beta ct$, I_c can be expressed as [3]

$$\begin{aligned}I_c(\theta,\phi,t^*) = {} & I_c(\theta_0,\phi_0,t^*)\exp(-\beta ds_o)\times\delta(\theta-\theta_0)\delta(\phi-\phi_0)\\ & \times\left[H\{t^*-\beta ds_o\}-H\left\{(t^*-\beta ds_o)-t_p^*\right\}\right]\end{aligned} \tag{7}$$

where H is the Heaviside function and δ is the Dirac-delta function.