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Effect of thermal relaxation time on heat transfer in a two layer composite system of living tissues☆

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ABSTRACT

In this paper, we have numerically studied the heat transfer in a composite system of living tissues by taking into consideration the effect of thermal relaxation time. The composite system of living tissues comprises two layers in which the heat transfer takes place in one layer due to pure diffusion, while in the other layer due to both perfusion and diffusion. The modified Pennes bioheat equation is considered by introducing a thermal relaxation time which is the ratio of the thermal diffusion to the square of the heat propagation velocity in the medium. We assume that heat mainly propagates in the direction perpendicular to the skin surface. The problem is solved numerically by developing the Crank–Nicolson implicit finite difference scheme. With an aim to validate our numerical results, a comparison has been made with the previous solution and shows excellent agreement. The study shows that the relaxation time has a significant impact on the temperature distribution in a two layer composite system of living tissues.

1. Introduction

Heat transfer in a composite system of living tissues is a very important process because of its many engineering applications. Hyperthermia treatment in cancer therapy is one of the most fundamental issues within biological tissues [1,2]. It has also other therapeutic applications involving rapid heating or cooling of living organs [3]. In most of the studies, bioheat transfer [2,4–7] has been modeled using the Pennes bioheat transfer equation based on the assumption that heat transfer takes place between the tissue and blood in capillaries. During the heat exchange between the tissue and the micro-capillary network (subcutaneous), perfusion plays an important role to bring the tissue temperature to its body core temperature. But there are some situations such as near the skin surface (dermis and epidermis) where no perfusion takes place. In this situation heat transfer is governed by pure diffusion. With this end in view, Becker [8] examined the heat conduction problem into a living perfuse and non-perfuse two layer composite system. In this study he modeled the problem based on the Pennes bioheat equation in the perfusion and diffusion layer, whereas a simple diffusion model was used in the non-perfusion i.e., in the pure diffusion layer. The problem has been solved analytically by using the separation of variables technique. However, the abovementioned study was restricted to the consideration of thermal relaxation time. Because the diffusion theory always produces a decayed temperature in time due to the presence of first order time derivative in the diffusion equation. Therefore, the thermal wave model introduces a relaxation time which results in a heat flux vector to respond to the thermal disturbances [6,9]. Moreover the delayed response of the temperature is due to the micro-structural interactions between the phonons and electrons on the microscopic level [10]. Lor and Chu [11] analyzed how interface resistance affects heat transfer in a two layered composite media, wherein they considered a simple hyperbolic heat conduction equation based on the thermal wave model. Their study is restricted to the consideration of blood perfusion phenomena in the depth tissue level. Shih et al. [12] examined the effects of pulsatile blood flow in a thermally significant blood vessel by considering the effective thermal conductivity of tumor tissues and thermal relaxation time in solid tissues of the temperature distribution during thermal treatment. Rodrigues et al. [13] carried out the analytical solution of a one dimensional bioheat transfer equation in a multilayer region with spatially dependent heat sources. They used different heat source terms to simulate the heating in a tumor and surrounding tissue, induced during a magnetic fluid hyperthermia technique. They developed their model based on the Pennes bioheat transfer equation, but without the corresponding thermal wave theory.

Owing to the abovementioned discussions, we have numerically studied the bioheat transfer in a two-layer composite system in the presence of thermal relaxation time using the thermal wave theory [14]. The governing equation based on the one dimensional spatially modified Pennes bioheat transfer equation subject to the appropriate boundary conditions is solved using an implicit finite difference scheme. The computed results are presented graphically for the dimensionless temperature with varying time and space (skin surface to the depth of the tissue) for different values of the thermal relaxation time (τ), the Biot number and blood perfusion rate.

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Nomenclature

C	Specific heat of tissue (J/kg °C)
C_b	Specific heat of blood (J/kg °C)
K	Thermal conductivity (W/m °C)
L	Length of the tissue layer (m)
q_m	Metabolic heat generation (J/m^3)
Q_m	Dimensionless heat generation: $Q_m = q_m L_2^2/k_2(T_\infty - T_b)$
t^*	Time (s)
t	Dimensionless time: $t = t^*\alpha_1/L_1^2$
T	Temperature of tissue (°C)
T_b	Core arterial blood temperature (°C)
T_∞	Ambient temperature (°C)
W_b	Perfusion rate of blood (kg/m^3 s)
W	Dimensionless perfusion: $W = \sqrt{W_b C_b \rho_b L_2^2/k_2}$
x^*	Dimensional spatial coordinate (m)
x	Dimensionless spatial coordinate: $x_i = x_i^*/L_i$
f	Initial condition
F	Initial condition
h	Convection heat transfer coefficient
H	Biot number: $H = hL_1/k_1$

Greek symbols

α	Thermal diffusivity: $\alpha_i = k_i/\rho c$
θ	Dimensionless temperature: $\theta_i = (T_i - T_b)/(T_\infty - T_b)$
κ	Dimensionless ratio of conduction resistances: $\kappa = \frac{(k_2/L_2)}{(k_1/L_1)}$
δ	Dimensionless ratio of Fourier numbers: $\delta = \frac{(\alpha_2/L_2^2)}{(\alpha_1/L_1^2)}$
ρ	Density (kg/m^3)
τ^*	Relaxation time (s)
τ	Dimensionless relaxation time: $\tau = \tau^*\alpha_1/L_1^2$
ω^*	Heating frequency
ω	Dimensionless heating frequency: $\omega = \frac{\omega^* L_1^2}{\alpha_1}$

2. Geometrical configuration and problem formulation

Let us consider a composite system of living tissues divided into two layers. The first layer is considered near the skin surface up to the tissue level of epidermis and dermis region, where the temperature distribution is assumed to be pure diffusion. The second layer is considered as depth of the tissue layer comprising the subcutaneous region in which the temperature distribution is maintained on the basis of the principle of both diffusion and perfusion. In the geometrical model (cf. Fig. 1) the length of the first layer is denoted by L_1 and that of the second layer is L_2. The spatial coordinate for both the layers is individually represented by x_1^* and x_2^* respectively along the X-axis. The interface between two layers is represented by the coordinate ($x_1^* = L_1$ and $x_2^* = 0$). A convective heat transfer with convection coefficient h and ambient temperature T_∞ is specified at the outer surface ($x_1^* = 0$) of the first layer. The second layer consists of perfuse tissue and experiences blood perfusion aided diffusion. Therefore the temperature at the other surface ($x_2^* = L_2$) is prescribed as arterial or blood temperature T_b. Let us introduce the thermal relaxation time τ^* (ratio of the coefficient of diffusivity to the square of the speed of the propagation of thermal wave) based on the thermal wave model in both the layers.

The one dimensional energy equation for the non-perfuse tissue in the first layer is governed by the diffusion equation as

$$\rho C\left(\tau^*\frac{\partial^2 T_1^*}{\partial t^{*2}} + \frac{\partial T_1^*}{\partial t^*}\right) = k_1\frac{\partial^2 T_1^*}{\partial x_1^{*2}};\quad 0 \le x_1^* \le L_1. \tag{1}$$

The one dimensional energy equation for the perfusion aided diffusion equation in the second layer is governed by the modified Pennes bioheat transfer equation [15] and is given by

$$\rho C\left(\tau^*\frac{\partial^2 T_2^*}{\partial t^{*2}} + \frac{\partial T_2^*}{\partial t^*}\right) = k_2\frac{\partial^2 T_2^*}{\partial x_2^{*2}} + \rho_b W_b C_b (T_b - T_2^*) + q_m - \tau^*\rho_b W_b C_b\frac{\partial T_2^*}{\partial t^*};\quad 0 \le x_2^* \le L_2, \tag{2}$$

where T_1^* and T_2^* are the temperature of the tissue for the first and second layers respectively, ρ is the density of the tissue, C is the specific heat of the tissue, k_1 is the thermal conductivity of the tissue in the first layer, k_2 is the thermal conductivity of the tissue in the second layer, ρ_b, C_b and W_b are respectively the density, specific heat and perfusion rate of blood and q_m represents the metabolic heat source.

It is noted that when $\tau^* = 0$, then Eq. (1) simply reduces to the heat conduction diffusion equation, while Eq. (2) represents the one dimensional Pennes bioheat transfer equation.

The boundary condition at the skin surface ($x_1^* = 0$) in terms of heat flux (cf. Fig. 1) is taken as

$$-k_1\frac{\partial T_1^*(0,t^*)}{\partial x_1^*} = h\left[T_b - T^*(0,t^*)\right]\sin(\omega^* t^*) \tag{3}$$

where h is the heat convection coefficient at the skin surface. The boundary condition at body core ($x_2^* = L_2$) temperature is taken as the same as blood temperature, i.e.,

$$T_2^*(x_2^* = L_2, t^*) = T_b. \tag{4}$$

The continuity boundary conditions at the interface between two layers ($x_1^* = L_1$ and $x_2^* = 0$) are given by

$$T_1^*(x_1^* = L_1, t^*) = T_2^*(x_2^* = 0, t^*) \tag{5}$$

and

$$k_1\frac{\partial T_1^*(L_1,t^*)}{\partial x_1^*} = k_2\frac{\partial T_2^*(0,t^*)}{\partial x_2^*}. \tag{6}$$

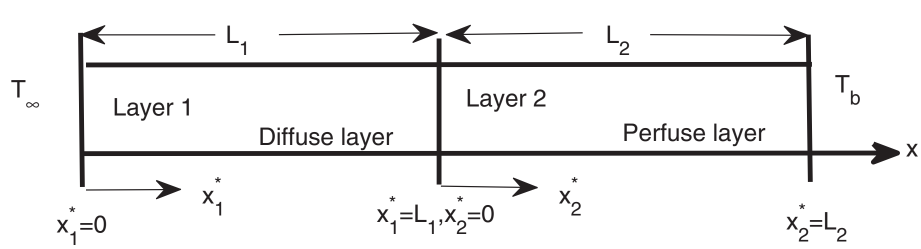

Fig. 1. A geometrical sketch of the problem.