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On axisymmetric heat conduction problem for FGM layer on homogeneous substrate☆

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ABSTRACT

The paper deals with the problem of heat conduction in the FGM layer with heat conductivity coefficient dependent on depth from boundary surface. The non-homogeneous layer is ideally bounded with homogeneous half-space with constant heat conductivity coefficient. The boundary plane is heated by: a) given temperature as a function of radius *r*, or b) given heat flux as a function of radius *r*. The Hankel transform method is applied to obtain a solution of formulated problem. The influence of thermal and geometric properties of FGM layer on temperature distributions in the considered bodies was investigated.

1. Introduction

Modern composite materials with functional graded properties (FGM) have a complex internal structure, due to the fact that they are composed of several different materials. The thermo-mechanical properties of the composite depend on the components included in such materials. There are two common types of FGM materials: with the continuous or discrete changing of the material properties. An example of FGM materials with the continuous changing of the material properties is the so-called FGM gradient zone created during the manufacturing process between the coating and the substrate component obtained under the action of temperature. Discrete changes in the properties are achieved by multilayer composites, which are very popular in technical applications. In the case of coatings with discrete changing of material properties the literature presented some models allowing the description ofa body by averaged technique [1–6]. However, this leads to a partial differential equations with variable coefficients and the solutions are possible in specific cases.

For the case of the so-called gradient zone resulting from the heat treatment process there is a continuous changing of mechanical properties of the material with the depth. In the literature it is known as an approach to such bodies, namely core material whichis treated as a homogeneous medium, and the transition zone from its surface is described by non-homogeneous materials with functional graded properties.

In many papers the researchers are concerned with the thermal and residual stress analysis. The FGM material has many applications in engineering construction and the heat conduction characteristics play a very important role in thermal and residual stress considerations [7,8]. These papers deal with exact solution of axisymmetric stationary conduction problem.

Contact problems of thermo-elasticity taking into account heat generation for graded half-space were considered in [9,10] (three-dimensional and axisymmetric problem) and [11] (two-dimensional problem). Thermal stresses in the FGM layer analyzed in the framework of the theory of plane state strain are studied by [12] and [13]. In all papers the thermo-mechanical properties are described by exponential function of distance to the surface of the body.

A special type gradient coating used in practice is the one with a periodic multilayer structure ([14,15]). This coating is composed of two homogeneous layers repeated periodically.

The paper [16] presents the newly proposed method using the proper transformation of variable, the Laplace transformation and the perturbation method of one-dimensional heat conduction problem in an FGM plate. The next paper [17] is devoted to analysis of the one-dimensional temperature distribution in an FGM strip using multi-layered approaches.

Some analytical solution of the problem of one-dimensional transient heat conduction in a material where the thermal conductivity coefficient is described by a linear function but the thermal diffusivity is treated as a constant is presented in [18]. The matrix method for heat conduction in circular cylinders of functionally graded materials and laminated composites was applied in paper [15]. Some numerical methods are used to describe the temperature distributions in materials with functional gradation properties (for example, [19, 20] and [21]).

The three-dimensional axisymmetric problem of thermal conduction of functionally graded circular plate, in which upper and lower boundary surfaces were kept in constant difference temperature was considered in [22]. Material properties were taken to be arbitrary distribution functions of the thickness. In this paper ordinary differential equations with variable coefficients ODE due to arbitrary distribution of

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Nomenclature

h	— thickness of layer [m];
a	— radius of heating zone [m];
$\lambda_1(z)$	— coefficient of heat conductivities of the FGM layer [W/(mK)];
λ_0	— coefficient of heat conductivities of the homogeneous half-space [W/(mK)];
T_0	— the temperature in the half-space [K];
T_1	— the temperature in the functionally graded layer (FGM);
q	— heat flux vector [W].

material properties along thickness coordinate was solved by the Peano–Baker series. The exact solution for three-dimensional axisymmetric problem for functionally graded circular plate obtained by variable separation method is presented in [23].

Namely, the axisymmetric problem of heat conduction in the non-homogeneous body composed of the FGM layer and homogeneous half-space is considered. It is assumed that the upper surfaces are heated by: 1) given temperature dependent on the radius, or 2) given heat flux dependent on the radius. Moreover the ideal thermal contact of the FGM layer and the homogeneous substrate is taken into account. The heat conductivity coefficients are assumed to be a power function of depth from the boundary surface. The considered problem is axisymmetric and independent of time. The Hankel transformmethod is applied to solve the considered problem.

The presented paper is one of continuity of research presented in [22,23] and obtained by Hankel transform method and two cases of heating on the upper boundary surface are considered: 1) given temperature dependent on the radius; and 2) given heat flux dependent on the radius.

2. The formulation and solution of the problem

The axisymmetric problem of heat conduction for FGM layer resting on the homogeneous space is considered. The analyzed problem is solved by using the cylindrical coordinate system (r, φ, z), where the z-axis is perpendicular to the lower boundary plane of the layer (see Fig. 1). It was assumed that $T_1(r, z)$ is the temperature in the functionally graded layer (FGM), and $T_0(r, z)$ is the temperature in the half-space.

Half-space is covered by a material with functional graded properties, and the coefficient of heat conductivity varies along with the depth

$$\lambda_1(z) = \lambda^*(c+z)^{\alpha}, \tag{1}$$

where λ^*, c, α are given constants.

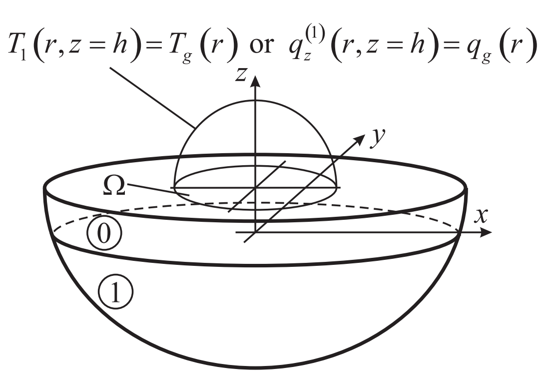

Fig. 1. The scheme of considered problem.

On the upper boundary surface two cases of heating conditions are assumed, namely a temperature, or a known heat flux dependent on the radius in the form:

- Given temperature

$$T_1(r, z=h) = T_g(r), \tag{2}$$

- Given heat flux

$$q_z^{(1)}(r, z=h) = q_g(r) \tag{3}$$

where $T_g(\cdot)$, $q_g(\cdot)$ are given functions.

The thermal properties of half-space are described by a heat conductivity coefficient $\lambda_0 = const$. In addition, the assumed ideal thermal contact between the layer and the half-space is assumed, which can be written as follows:

$$\begin{aligned} &\lim_{z\to 0^+} T_1(r,z) = \lim_{z\to 0^-} T_0(r,z), \\ &\lim_{z\to 0^+} \lambda_1(z)\frac{\partial T_1(r,z)}{\partial z} = \lim_{z\to 0^-} \lambda_0 \frac{\partial T_0(r,z)}{\partial z}. \end{aligned} \tag{4}$$

Moreover, radiation conditions at infinityare taken into account. Equation of heat conduction takes the following form

- for homogeneous half-space

$$\frac{\partial^2 T_0}{\partial r^2} + \frac{1}{r}\frac{\partial T_0}{\partial r} + \frac{\partial^2 T_0}{\partial z^2} = 0, z < 0, r > 0, \tag{5}$$

- FGM layer

$$\frac{\partial^2 T_1}{\partial r^2} + \frac{1}{r}\frac{\partial T_1}{\partial r} + \frac{\partial^2 T_1}{\partial z^2} + \frac{1}{\lambda_1(z)}\frac{\partial \lambda_1(z)}{\partial z}\frac{\partial T_1}{\partial z} = 0, 0 < z < h, r > 0. \tag{6}$$

Components of the heat flux vector:

- in the homogeneous half-space

$$\vec{q}_0(r, z) \equiv \left(q_r^{(0)}, 0, q_z^{(0)}\right) = \left(\lambda_0\frac{\partial T_0}{\partial r}, 0, \lambda_0\frac{\partial T_0}{\partial z}\right), \tag{7}$$

- in the FGM layer

$$\vec{q}_1(r, z) \equiv \left(q_r^{(1)}, 0, q_z^{(1)}\right) = \left(\lambda_1(z)\frac{\partial T_1}{\partial r}, 0, \lambda_1(z)\frac{\partial T_1}{\partial z}\right). \tag{8}$$

The solution of the above formulated problem are calculated by using the Hankel transform method, which is denoted by

$$\tilde{f}(s, z) = \int_0^{\infty} f(r, z) r J_0(sr) dr. \tag{9}$$

Using Eqs. (9) to (5) and (6) we obtain the ordinary differential equations;

- for the homogeneous half-space

$$\frac{d^2\tilde{T}_0}{dz^2} - s^2\tilde{T}_0 = 0, \tag{10}$$