International Communications in Heat and Mass Transfer 57 (2014) 193–199

Contents lists available at ScienceDirect

International Communications in Heat and Mass Transfer

journal homepage: www.elsevier.com/locate/ichmt

Numerical thermal analysis of skin tissue using parabolic and hyperbolic approaches ☆

G.R. Ströher *, G.L. Ströher

Coordination of Chemical Engineering, Federal University of Technology — Paraná, Apucarana, Paraná, Brazil

ARTICLE INFO

Available online 5 August 2014

Keywords:
Numerical thermal analysis
Skin tissue
Classical Fourier law
Non-Fourier

ABSTRACT

The understanding of heat transfer in skin tissue is of utmost significance in various areas. Especially in medicine is required a precise determination of the temperature distribution for not thermally damaging healthy tissue when any region of the body is subjected to a heat treatment. The accuracy in predicting temperatures is linked to the use of adequate thermal and numerical methods. In this way, this study presents the results of a two-dimensional model to calculate transient temperature and burn injury distributions in skin tissue. Heat transfer was modeled using the Pennes' thermal model, and the mechanism of heat conduction assessed through two different approaches, classical Fourier law and non-Fourier law, characterized mathematically as parabolic and hyperbolic, respectively. The numerical solutions of the two approaches were compared to analytical solutions reported in the literature, as well as are shown various numerical results under various conditions to evaluate the differences between the two approaches to predict the temperature distribution and thermal damage.

1. Introduction

The use of heat for therapeutic purposes dates back to ancient times, Egyptians, Romans, Greeks and other ancient peoples used it to treat various types of diseases and muscle problems. The moxibustion, therapy that uses the heat generated by the burning of *Artemisia vulgaris* to stimulate acupuncture points is a representative of therapeutic modality in traditional medicine for more than 2500 years [1].

Heat increases the temperature of biological tissues and generally promotes changes in physical properties thereof, and depending on the intensity of the heat source may cause: vasodilation, venous congestion in injured tissues, increased capillary permeability, muscle relaxation and many other effects. Currently, heat is used in various medical treatments, especially for cauterizing tissue, treating degenerative diseases of joints, arthritis, muscle tension and pain.

Heat is also an important tool in oncology, administered in different ways. Generally, the heat treatment is applied as a supplement to a conventional treatment (particularly radiation therapy and chemotherapy). Lima et al. [2] mention that inoperable duodenal tumors can be irradiated with laser sources endoscopically. The heat from the laser elevates local temperature in order to kill the cancer cells without, however, causing thermal damage to healthy surrounding region.

It is extremely inevitable to understand the temperature rise activities occurring in biological tissues during hyperthermia treatment. Especially, the temperature distribution inside as well as outside the target region must be known as a function of the exposure time in order to provide a level of therapeutic temperature and on the other hand, to avoid overheating and damaging of the surrounding healthy tissue (Gupta et al. [3]).

The accuracy of the calculation of temperatures is linked to the use of adequate phenomenological models and numerical methods. Fourier's law, $\mathbf{q} = -k\nabla T$, is one of the most widely used models for heat conduction. Although this model provides some nonphysical results, for decades, Fourier's law has relative agreement between theory and experimental data. Among the problems of Fourier's law, the main one is that the model implies an infinite speed of heat propagation (Mishra et al. [4]; Mishra et al. [5]; Ali and Zhang [6]), which is physically inadmissible.

Basically, Fourier's law has the property that a heat signal applied to a face of a body is immediately felt in all parts of the body, so it is implicitly assumed that the propagation speed is infinite. This anomaly has been supposedly overcome by the modified Fourier equation, which is usually attributed to Cattaneo [7] and Vernotee [8], this modification includes a term that recognizes the finite speed of heat signals. Through the kinetic theory and Boltzmann equations for rarefied gases, those authors proposed $\tau_q\, \partial \mathbf{q}/\partial t + \mathbf{q} = -k\nabla T$, where τ_q is the relaxation time defined as the time necessary for storage of the thermal energy required for the propagative transfer to an element in thermal contact.

As the thermal diffusivity, the relaxation time is a function of the material and usually the numerical value is small, 10^{-10} s for gases and 10^{-12} s for liquids, Kaminski [9]. As the relaxation time tends to zero, the modified Fourier equation approaches the classical Fourier's law. However, for inhomogeneous materials, Kaminski [9] found thermal relaxation times above 10 s and Luikov [9] in the range

☆ Communicated by W.J. Minkowycz.

* Corresponding author.
E-mail addresses: gylles@utfpr.edu.br (G.R. Ströher), gisely@utfpr.edu.br (G.L. Ströher).

http://dx.doi.org/10.1016/j.icheatmasstransfer.2014.07.026
0735-1933/

0.001–1000 s, further data for thermal relaxation time can be obtained in Sharma [10].

The human skin is a heterogeneous material primarily composed of cells, fibrillar elements and liquids which are distributed in layers. Mitra et al. [11] reported experimental results on processed meat with a thermal relaxation value of about 15 s, which is much higher than reported for metals. Vedavarz et al. [12] observed for some biological tissues at room temperature τ_q between 1 and 1000 s, Matsunaga [13] obtained for some organs of pigs the range of τ_q 2.91–21.31 s. Because of the relatively high value of the thermal relaxation time for biological tissues, the thermal behavior provided by the modified Fourier equation has been experimentally observed and thus attracting increasingly more attention (Xu et al. [14]). Importantly, as noted by many authors, it is possible to design boundary conditions such that Fourier Modified provides heat that would appear to be moving from a cold to a hot point, in violation of the second law of thermodynamics (Ali and Zhang [6]), however, as a limiting approximation both models are subject to a nonphysical description of certain phenomena, like the well-known infinite propagation speed and the temperature overshoot phenomenon (Taitel [15]).

Many studies compared the thermal responses to biological systems using Fourier's law and the modified Fourier's law, also known as Non-Fourier. As described in the next section, the option of using the Fourier or Non-Fourier law in the Pennes [16] equation provides models mathematically characterized as parabolic and hyperbolic, respectively. In the hyperbolic model, the equation of heat diffusion becomes second order in time and space and liable to fictional numerical oscillations at discontinuities (Mishra and Sahai [5]). Most studies applying the parabolic or hyperbolic approaches are limited to one-dimensional cases Jiang [17], Shih et al. [18], Xu et al. [14], and Ahmadikia et al. [19], however in many situations it is necessary to know the temperature distribution when a region is subjected to heat flow as in the case of explosions or some thermal medical treatment. In this context, this study presents the results from the development of a two-dimensional unsteady solver, implementing parabolic and hyperbolic approaches. The numerical solution given by the solver was compared with diverse one-dimensional solutions available in the literature and predictions provided by parabolic and hyperbolic approaches were compared for one- and two-dimensional cases.

2. Heat transfer model

2.1. Parabolic approach

This model for bioheat transfer, the Pennes equation [16] is well known. In this model the conduction term is based on Fourier's law.

$$q\left(\vec{r},t\right) = -k\nabla T\left(\vec{r},t\right) \tag{1}$$

where q is the heat flux vector representing heat flow per unit time, per unit area; k is the thermal conductivity, ∇T is the temperature gradient, $\vec{r}$ is the position vector, and t is the time.

The Pennes equation for modeling skin tissue heat can be obtained combining Eq. (1) with the general bioheat transfer equation:

$$\rho_t c_t \frac{\partial T}{\partial t} = -\nabla q + \varpi_b \rho_b c_b (T_a - T) + q_{met} + q_{ext} \tag{2}$$

where, ρ_t, c_t, and k are the density, specific heat and thermal conductivity of skin tissue, respectively; ϖ_b is the blood perfusion rate; ρ_b and c_b are the density and specific heat of the blood, respectively; T_a and T are the blood and skin tissue temperatures, respectively; q_{met} is the metabolic heat generation in skin tissue and q_{ext} is the heat generated by other heat sources. In this present work, we considered q_{ext} as zero for all simulations.

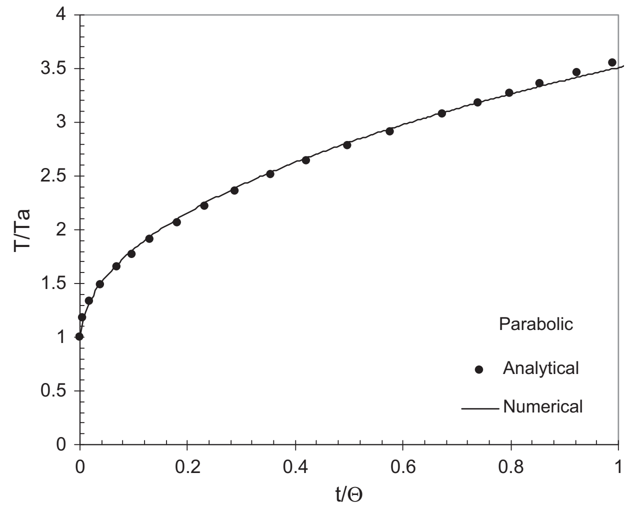

Fig. 1. Comparison between analytical and numerical profiles for constant heat flux with parabolic model.

By combining Eqs. (1) and (2):

$$\rho_t c_t \frac{\partial T}{\partial t} = k\nabla^2 T + \varpi_b \rho_b c_b (T_a - T) + q_{met} + q_{ext}. \tag{3}$$

As aforementioned, the Pennes equation is based on the classic Fourier's law for heating, which assumes that the propagation speed of any temperature disturbance or thermal wave is infinite.

2.2. Hyperbolic approach

Cattaneo [7] and Vernott [8] used the concept of a finite heat propagation speed, and formulated a modified unsteady heat conduction equation, which is a linear extension of the unsteady Fourier equation:

$$q\left(\vec{r},t+\tau_q\right) = -k\nabla T\left(\vec{r},t\right) \tag{4}$$

where, $\tau_q = \alpha/C^2{}_t$ is defined as the thermal relaxation time, α stands for the thermal diffusivity, and C is the thermal wave speed in the medium [20].

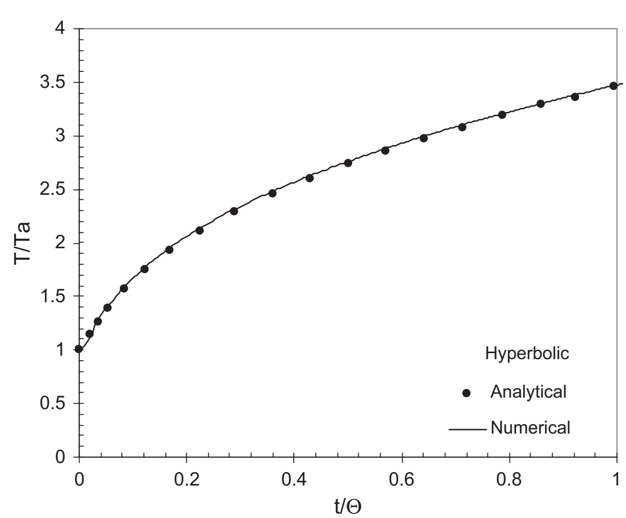

Fig. 2. Comparison between analytical and numerical profiles for constant heat flux with hyperbolic model.