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Temperature in thermally nonlinear pad–disk brake system ☆

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ABSTRACT

The influence of the thermal sensitivity of pad and disk materials on temperature at braking is under investigation. A mathematical model of process of frictional heating in a pad–disk brake system, which takes into account the temperature-sensitive materials, is proposed. The basic element of this model is the thermal problem of friction—a one-dimensional boundary-value heat conduction problem with temperature-dependent thermal conductivity and specific heat. Contrary to the prior studies of authors, where a simple nonlinearity was considered, in this article the arbitrary nonlinearity of the thermophysical properties of materials is studied. The solution of a nonlinear boundary-value heat conduction problem is obtained by the method of successive approximations. The numerical analysis of temperature is executed for some materials of a pad and a disk with and without taking into account their thermal sensitivity.

1. Introduction

Reviews of analytical and numerical methods of solving boundary-value problems of heat conduction for materials with temperature-dependent thermal properties (thermosensitive materials) are presented in papers [1–3]. It was noticed that such materials can be divided into two categories. The first one includes materials with a simple nonlinearity, i.e. such in which the coefficients of thermal conductivity and specific heat are temperature dependent, and their ratio – coefficient of thermal diffusivity – is constant [4–6]. The thermosensitive materials without such a property make the second category of materials with arbitrary nonlinearity [7–9]. Therefore, various methods of solving the corresponding thermal problems of friction have been developed. For materials with a simple nonlinearity this is the step-by-step linearization method by means of spline-approximation or linearizing parameters [10–12]. In case of thermosensitive materials with arbitrary nonlinearity, the use of iterative methods is effective, in particular the method of successive approximations [13]. A solution to the nonstationary heat conduction problem of friction for two semi-infinite bodies in the case of sliding with a constant speed was obtained by this method in article [14]. The solution of the thermal problem of friction during braking with a constant retardation, when thermosensitive materials of a pad and a disk have simple nonlinearity, was obtained by means of the step-by-step linearization method in article [15]. In this article the solution to the same problem is found in the case of arbitrary nonlinearity of materials of a pad and a disk.

2. Statement of the problem

Let two semi-spaces be compressed at infinity with a constant pressure p_0 in a direction parallel to the z -axis of a Cartesian coordinate system $Oxyz$ (Fig. 1). In the initial time moment $t = 0$ the semi-spaces begin the relative sliding in the positive direction of y -axis with a speed $V(t) = V_0(1 - t/t_s), 0 \le t \le t_s$, where V_0 is the initial velocity, and t_s is the stop time. On the contact surface $z = 0$, due to the friction, the heat is generated and the bodies are heated. It is assumed, that the sum of the intensities of heat fluxes, directed along the normal to the surface of contact inside the semi-spaces, is equal to the specific power of friction $q(t) = fV(t)p_0$ [16], where f is the coefficient of friction. The thermal contact of the bodies is imperfect—through the contact surface the heat transfer takes place with a constant coefficient of thermal conductivity of contact h [17,18]. The coefficients of heat conduction K_l and specific heat c_l of materials are temperature dependent:

$$K_l(T) = K_{l,0}K_l^*(T), c_l(T) = c_{l,0}c_l^*(T), l = 1, 2, \tag{1}$$

where

$$K_{l,0} \equiv K_l(T_0), c_{l,0} \equiv c_l(T_0), \tag{2}$$

and dimensionless functions $K_l^*(T)$ and $c_l^*(T)$ can be written in polynomial form:

$$K_l^*(T) = 1 + \sum_{n=1}^{N_l} a_{l,n}\Delta T^n, c_l^*(T) = 1 + \sum_{n=1}^{M_l} b_{l,n}\Delta T^n, l = 1, 2, \tag{3}$$

$\Delta T = T - T_0$ is the temperature increase. We also assume that densities of materials ρ_l, $l = 1,2$ are constant.

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Nomenclature

a	characteristic dimension;
Bi	Biot number;
c	specific heat;
c_0	specific heat at an initial temperature;
$\mathrm{erf}(x)$	Gauss error function;
f	friction coefficient;
$H(\cdot)$	Heaviside step function;
h	coefficient of thermal conductivity of contact;
K	coefficient of thermal conductivity;
K_0	coefficient of thermal conductivity at an initial temperature;
k	coefficient of thermal diffusivity;
p_0	pressure;
q	specific power of friction;
T	temperature;
T_0	initial temperature;
T^*	dimensionless temperature;
t	time;
t_s	braking time;
V	sliding speed;
z	spatial coordinate;
Θ	Kirchhoff's variable;
ρ	specific density;
τ	dimensionless time;
τ_s	dimensionless braking time;
ζ	dimensionless spatial coordinate;

Subscripts:

1	the upper semi-space,
2	the bottom semi-space.

Here and further all values referring to the upper and lower semi-spaces will have subscripts 1 and 2, respectively.

Taking into account the assumptions mentioned above, we find the distribution of transient temperature field $T(z,t)$ in semi-spaces from solution of the following nonlinear boundary-value heat conduction problem:

$$\frac{\partial}{\partial\zeta}\left[K_1^*(T^*)\frac{\partial T^*}{\partial\zeta}\right]=\frac{c_1^*(T^*)}{k_0^*}\frac{\partial T^*}{\partial\tau},\zeta>0,0<\tau\le\tau_s, \tag{4}$$

$$\frac{\partial}{\partial\zeta}\left[K_2^*(T^*)\frac{\partial T^*}{\partial\zeta}\right]=c_2^*(T^*)\frac{\partial T^*}{\partial\tau},\zeta<0,0<\tau\le\tau_s, \tag{5}$$

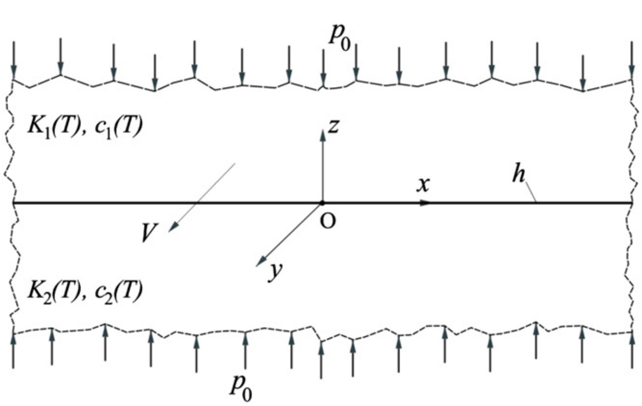

Fig. 1. Scheme of the problem.

$$K_2^*(T^*)\left.\frac{\partial T^*}{\partial\zeta}\right|_{\zeta=0^-}-K_0^*K_1^*(T^*)\left.\frac{\partial T^*}{\partial\zeta}\right|_{\zeta=0^+}=q^*(\tau),0<\tau\le\tau_s, \tag{6}$$

$$K_2^*(T^*)\left.\frac{\partial T^*}{\partial\zeta}\right|_{\zeta=0^-}+K_0^*K_1^*(T^*)\left.\frac{\partial T^*}{\partial\zeta}\right|_{\zeta=0^+}=Bi\left[T^*\left(0^+,\tau\right)-T^*(0^-,\tau)\right],0<\tau\le\tau_s, \tag{7}$$

$$T^*(\zeta,\tau)\to T_0^*,|\zeta|\to\infty,0<\tau\le\tau_s, \tag{8}$$

$$T^*(\zeta,0)=T_0^*,|\zeta|<\infty, \tag{9}$$

where

$$q^*(\tau)=1-\frac{\tau}{\tau_s},0\le\tau\le\tau_s, \tag{10}$$

$$K_l^*(T^*)=1+\sum_{n=1}^{N_l}A_{l,n}(\Delta T^*)^n,A_{l,n}=a_{l,n}(T_a)^n,l=1,2, \tag{11}$$

$$c_l^*(T^*)=1+\sum_{n=1}^{M_l}B_{l,n}(\Delta T^*)^n,B_{l,n}=b_{l,n}(T_a)^n,l=1,2, \tag{12}$$

$$\zeta=\frac{z}{a},\tau=\frac{k_2t}{a^2},K_0^*=\frac{K_{1,0}}{K_{2,0}},k_0^*=\frac{k_{1,0}}{k_{2,0}},Bi=\frac{ha}{K_{2,0}}, \tag{13}$$

$$T_a=\frac{q_0a}{K_{2,0}},T^*=\frac{T}{T_a},T_0^*=\frac{T_0}{T_a}, \tag{14}$$

and $a=1.73\sqrt{k_{2,0}t_s}$ is the effective depth of heat penetration—distance from the friction surface, on which the temperature is equal 5% of the maximal temperature on the surface of friction [19]; $q_0=fV_0p_0$ is the specific power of friction in the initial time moment, and $\Delta T^*=T^*-T_0^*$ is the dimensionless temperature increase.

3. Solution of the problem

Using the Kirchhoff transformation [20]

$$\Theta_l(\zeta,\tau)=\int_{T_0^*}^{T^*(\zeta,\tau)}K_l^*(T)dT,l=1,2, \tag{15}$$

the boundary-value problem (4)–(9) can be written as:

$$\frac{\partial^2\Theta_1}{\partial\zeta^2}=\frac{1}{k_0^*k_1^*(T^*)}\frac{\partial\Theta_1}{\partial\tau},\zeta>0,0<\tau\le\tau_s, \tag{16}$$

$$\frac{\partial^2\Theta_2}{\partial\zeta^2}=\frac{1}{k_2^*(T^*)}\frac{\partial\Theta_2}{\partial\tau},\zeta<0,0<\tau\le\tau_s, \tag{17}$$

$$\left.\frac{\partial\Theta_2}{\partial\zeta}\right|_{\zeta=0}-K_0^*\left.\frac{\partial\Theta_1}{\partial\zeta}\right|_{\zeta=0}=q^*(\tau),0<\tau\le\tau_s, \tag{18}$$