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Large eddy simulation of transition of free convection flow over an inclined upward facing heated plate☆

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ABSTRACT

Transition of free convection flow of air over an inclined heated surface is investigated numerically by using a large eddy simulation method. In particular, we focus on how inclination angle of an upward-facing heated plate affects flow transition. Special attention is paid to the development of the thermal boundary layer and the transition from the laminar to turbulent stage. Results show that the transition occurs early when the plate is moved from its vertical position due to the rapid growth of both the velocity and thermal boundary layers. As a consequence, the critical Grashof number drops. Effects of the inclination of plate on the turbulent velocity fluctuations are also investigated, and the predicted results are in very good agreement with various experimental data available in the literature.

1. Introduction

Transition of a thermal boundary layer over an inclined heated plate has received significant attention over the years due to its wide range of applications in engineering systems: such as transistors, mainframe computers, heat exchangers, solar energy collectors, and nuclear reactor cooling elements. Rich [1] carried out experiments of laminar free convection flow on a heated plate with its angular orientation varying from 0° to 45°, while later Kierkus [2] focused on two-dimensional analysis of laminar free convection and satisfactorily compared his results with the experimental data of Rich [1].

Hassan and Mohamed [3] measured the local heat transfer on an isothermal plate. The angle of inclination was varied from −90° to 90°, as the study particularly focused on the transition of flow on both the downward and upward facing heated surfaces. However, their results did not show any transition stage on the downward facing heated surface due to the relatively short length of the plate used in the experiment. For the same reason, they could not predict the transition stage in the vertical case. Nevertheless, the transition behaviour for the upward plate case was summarised, and correlation equations for important flow parameters were suggested at the transition region.

Tritton [4], on the other hand, used a fibre anemometer method to capture transition as well as the development of full turbulence on an isothermal plate. The experiment was performed in air and the Grashof number (ratio of buoyancy to viscous forces) on a vertical plate was recorded to be 9.26×10^6, which is much smaller than Grashof numbers found in other experiments. Tritton suggested that this behaviour was mainly due to the strong flow disturbances in his experimental set-up and possible uncertainties in the methods he used. Sparrow and Hussar [5] and Lloyd and Sparrow [6] established later a correlation between the inclination angle of a heated plate and the instability of the thermal boundary layer. Their results showed that the onset of transition was characterised by longitudinal vortices. Similarly, Gebhart [7] studied two different types of instability, thermal and hydrodynamic, in buoyancy induced flows. Their effects on the flow transition were investigated on a vertical, a horizontal and an inclined surface, respectively.

Later, Black and Norris [8] provided a method for flow visualisation and measured local heat transfer coefficient for the natural convection on an inclined isothermal plate. They particularly found that the thermal sub-layer contains thermal waves that traverse the heated plate and cause significant variations in the local heat transfer. Recently, natural convection over an upward facing inclined plate was examined by Komori et al. [9] and Kimura et al. [10], where the plates were kept under the heat flux thermal conditions in order to study the mechanism of transition from laminar to turbulent flow. The flow over the heated plate and the wall temperature were visualised by fluorescent paint and a liquid-crystal thermometry respectively. Komori et al. [9] showed that the separation of the boundary layer and the onset of streaks appear when the modified Rayleigh number exceeds a characteristic value. Nevertheless, the complete transition process was not fully examined.

Recently, Paul et al. [11] studied the effects of plate angle on the thermal boundary layer stability. They investigated the influence of high order effects of the boundary layer on the vortex instability of thermal boundary layer flow in a wedge-shaped domain formed by the heated and the outer boundary surface. Most recently, Alzwayi and Paul [12]

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Nomenclature

Roman symbols

Cs	Smagorinsky model constant
g	gravitational acceleration
Gr	Grashof number
L	length of the plate
n_x, n_y, n_z	number of nodes in the x, y and z directions respectively
Nu	Nusselt number
P	pressure
Pr	Prandtl number
S_{ij}	strain rate tensor
T	temperature
t	time
u_i or u, v, w	velocity components in the Cartesian coordinates
x_i or x, y, z	Cartesian coordinate directions

Greek symbols

θ	angle of inclination with respect to the vertical position
β	thermal expansion coefficient
Γ	diffusion coefficient for energy transport
Δ	filter width
μ	molecular viscosity of fluid
ρ	fluid density
τ_{ij}	subgrid scale stresses

Subscripts

a	air
c	critical
∞	ambient conditions
sgs	sub-grid scale
max	maximum
P	plate
rms	root mean squares

numerically investigated the transition of a thermal boundary layer in a vertical parallel plate channel and its dependence on plate width and temperature. In the context of a heated surface facing downward, Alzwayi and Paul [13] also studied the transition of free convection flow in an inclined parallel walled channel. They investigated the effects of inclination angle and width of the channel on transition. However, very little is known about the transition of flow developing on a heated surface facing upward and how it is affected by its inclination angle.

The aim of this paper is therefore to carry out an investigation on the transition phenomena of a free convection flow developing on an inclined heated plate by large eddy simulation (LES). The paper focus on how the flow and thermal fields jointly affect the transition process and the effects they have on the turbulent quantities such as fluctuating velocity components, which are generated after the flow transition.

2. Model geometry

A schematic drawing of the model geometry and computational domain with coordinate systems is given in Fig. 1. An isothermal heated plate, temperature of which is denoted by Tp, is exposed to the environment at an ambient temperature denoted by Ta. Driven by the buoyancy force, the air heated up by the hot wall flows along the heated surface and forms natural convection boundary and thermal layers. The length of the plate in the y direction ($L = L_y$) is 3 m. The free boundary opposite to the plate is placed at a distance equal to 3.5 times the thickness of the fully developed boundary layer at the downstream (which corresponds to $L_x \approx 1$ m). A periodic boundary condition is implemented in the

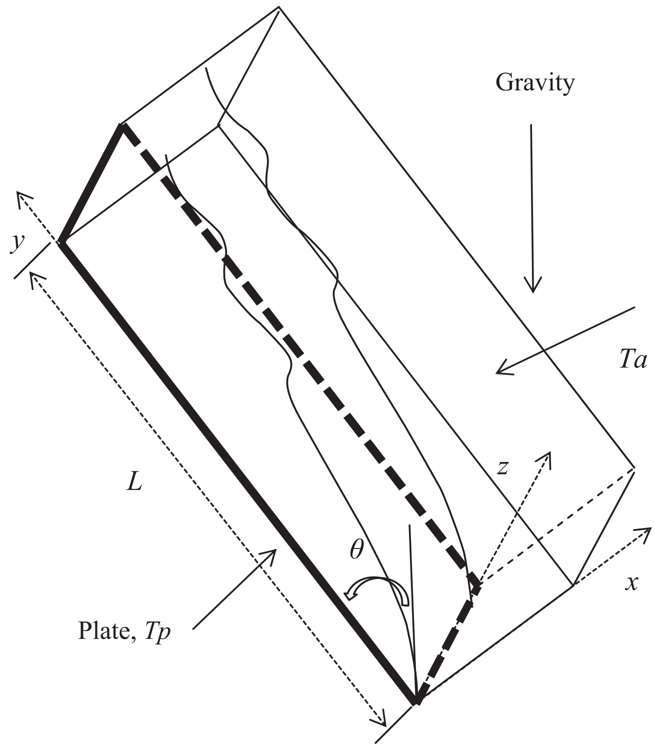

Fig. 1. Flow geometry with coordinate systems.

spanwise direction in the z-direction, with the boundaries separated 1.5 m.

3. Mathematical model

3.1. Governing equations

Free convection over the heated plate is governed by the three-dimensional unsteady-state Navier–Stokes equations together with the energy equation. The working fluid (air with $Pr = 0.71$) flow is considered to be Newtonian and all the physical properties of air are assumed to be constant. The filtered governing equations of flow and energy for large eddy simulations, which are subject to the Boussinesq approximation, take the following forms:

$$\frac{\partial \rho}{\partial t} + \frac{\partial \rho \overline{u}_i}{\partial x_i} = 0 \tag{1}$$

$$\frac{\partial \rho \overline{u}_i}{\partial t} + \frac{\partial \rho \overline{u}_j \overline{u}_i}{\partial x_j} = -\frac{\partial \overline{P}}{\partial x_i} + \frac{\partial}{\partial x_j}\left(\mu \frac{\partial \overline{u}_i}{\partial x_j}\right) - \frac{\partial \tau_{ij}}{\partial x_j} + \rho \overline{g}_i \tag{2}$$

$$\frac{\partial \rho \overline{T}}{\partial t} + \frac{\partial \rho \overline{u}_j \overline{T}}{\partial x_j} = \frac{\partial}{\partial x_j}\left(\Gamma \frac{\partial \overline{T}}{\partial x_j}\right) - \frac{\partial J_j^{sgs}}{\partial x_j} \tag{3}$$

where $\overline{g}_i = \left[g \sin\theta \beta\left(\overline{T} - \overline{T}_\infty\right), g \cos\theta \beta\left(\overline{T} - \overline{T}_\infty\right), 0\right]$, $\Gamma = \mu/Pr$ is the diffusion coefficient, x_j is the coordinate system and u_j is the corresponding velocity components. P is the pressure, ρ is the density, μ is the dynamic viscosity, and Pr is the Prandtl number of air. The spatial filter applied to obtain the governing LES equations separates the large scale (resolved scale) flow field from its small scale (sub-grid scale) [14]. The effects of the small scales appear in the sub-grid stress terms, defined as

$$\tau_{ij} = \rho \overline{u_i u_j} - \rho \overline{u}_i \overline{u}_j \tag{4}$$