International Communications in Heat and Mass Transfer 57 (2014) 347–352

Contents lists available at ScienceDirect

International Communications in Heat and Mass Transfer

journal homepage: www.elsevier.com/locate/ichmt

Empirical Nu–Ra–Fo relationships for natural convection in air-filled hemispherical enclosures. Isothermal and inclined disk with dome oriented downwards ☆

A. Baïri

University of Paris, Laboratoire Thermique Interfaces Environnement, LTIE-GTE, EA 4415, 50 Rue de Sèvres, F-92410 Ville d'Avray, France

ARTICLE INFO

Available online 16 August 2014

Keywords:
Nu–Ra–Fo correlations
Transient natural convection
Hemispherical cavity
Thermal engineering
Large Rayleigh number range
Dirichlet thermal boundary condition

ABSTRACT

Correlations of Nusselt–Rayleigh–Fourier type proposed in this work allow quantifying of the transient convective heat transfer occurring in air-filled hemispherical cavities. The disk, initially at ambient temperature, is suddenly heated and kept isothermal. Throughout the heating process, the dome is maintained at ambient temperature. The radius of the cavity, associated with the temperature difference imposed between the disk and the dome, involves a large Rayleigh number range, varying between 10^4 and 5×10^8. The disk can be inclined with respect to the horizontal plane at an angle varying between 90° (vertical disk) and 180° (horizontal disk with dome downwards) by steps of 10°. The numerical approach is based on the finite volume method. The proposed empirical relationships in transient regime are linked to the steady state Nusselt–Rayleigh ones. They are obtained via the analysis of results corresponding to many combinations of the Rayleigh number and the disk inclination angle. The relationships are new since the considered geometry associated with the inclinations of the isothermal disk has not been treated previously. They constitute an important tool for the thermal design of engineering systems involved as they allow determining the convective heat transfer during the transient regime. They can be applied in several fields such as nuclear technology, solar energy, security and safety electronics, building, domotics or aeronautics.

1. Introduction

The hemispherical enclosures whose base is the active hot plate are used in various engineering fields. Dimensions and temperature levels in the considered configuration can lead to large Rayleigh number values. This is the case in nuclear techniques where these geometries can provide the cover plants (EPR case with dome upwards) or the reactor basin (dome downwards). This is also the case in building. The upwards-facing dome has indeed inspired many thermal and civil engineers (Iken), as well as architects. This is highlighted by various monuments such as the domes of the Academy, the Pantheon, the Val-de-Grâce or the Hôtel des Invalides in Paris. This is also the case for igloos which have thermal and mechanical properties suitable for the environment in which they are installed. Large Rayleigh numbers also concern solar stills whose horizontal disk constituting the absorber is covered with a transparent dome in which the greenhouse effect takes place. Some solar thermal collectors are equipped with a hemispherical transparent cover, the absorber disk being installed on a horizontal or inclined plane. Moreover, solar energy studies include lower Rayleigh number configurations that are associated with moderate values of the radius and the temperature level (or heat flux). Components of global solar radiation are measured by means of pyranometers that are installed either horizontally or on an inclined plane depending on the intended application. Low Rayleigh numbers relate also to the safety and security field, where hemispherical enclosures are often used as sensors to control various devices. The geometry of the cavity is one of the physical parameters which most highly affect the flow within the cavity, and thus the natural convective heat transfer, in steady state and in transient regime. This was confirmed in several studies dealing with cavities of square [1–3], parallelogrammic [4–6], triangular [7], cylindrical [8], or trapezoidal [9] shape.

Most studies dealing with the hemispherical geometry are done in steady state and deal with the case of the horizontal and isothermal disk. The influence of the disk inclination angle (Fig. 1) on the natural convection within hemispherical cavities has been treated in [10], but with no quantification of the convective heat transfer. The cavity with horizontal and isothermal disk is treated in [11–15]. These works contain some elements allowing determining of the convective heat transfer for some specific Rayleigh ranges. Imposed heat flux on the disk (Neumann condition) applied to electronic technologies (Ania) is examined in [16] at steady state for angle inclinations $0 \le \alpha \le 90°$ and Rayleigh numbers $10^4 \le Ra_\varphi \le 5 \times 10^7$. Correlations of Nusselt–Rayleigh type contained in [16] are extended in [17] to $5 \times 10^7 \le Ra_\varphi \le 2.55 \times 10^{12}$.

☆ Communicated by W.J. Minkowycz.

E-mail addresses: abairi@u-paris10.fr, bairi.a@gmail.com.

http://dx.doi.org/10.1016/j.icheatmasstransfer.2014.08.016
0735-1933/

Nomenclature

a	thermal diffusivity ($m^2\ s^{-1}$)
$C_T(\alpha)$	exponent of Fo in $\overline{Nu} = \underline{\underline{Nu}}\, Fo^{C_T(\alpha)}$ for Dirichlet condition (–)
$\overline{C_T}(\alpha)$	average value of $C_T(\alpha)$
$C_\varphi(\alpha)$	exponent of Fo in $\overline{Nu}^{\varphi} = \underline{\underline{Nu}}^{\varphi} Fo^{C_\varphi(\alpha)}$ for Neumann condition (–)
C_p	specific heat at constant pressure ($J\ kg^{-1}\ K^{-1}$)
Fo	Fourier number (–)
g	gravity acceleration ($m\ s^{-2}$)
g_r, g_ϕ, g_θ	components of g in r,ϕ,θ directions respectively ($m\ s^{-2}$)
$k(\alpha)$	coefficient of the correlations $\underline{\underline{Nu}} = k(\alpha) Ra_T^{n(\alpha)}$ (-)
$k^{'}(\alpha)$	coefficient of the correlations $\underline{\underline{Nu}}^{\varphi} = k'(\alpha) Ra_\varphi^{n'(\alpha)}$ (-)
m	number of elements on the disk
$n(\alpha)$	exponent of Ra_T in the correlations $\underline{\underline{Nu}} = k(\alpha) Ra_T^{n(\alpha)}$ (-)
$n^{'}(\alpha)$	exponent of Ra_φ in the correlations $\underline{\underline{Nu}}^{\varphi} = k'(\alpha) Ra_\varphi^{n'(\alpha)}$ (-)
Nu_i	local transient Nusselt number on the disk (–)
$\overline{Nu}$	average transient Nusselt number on the disk for Dirichlet condition (–)
$\underline{\underline{Nu}}$	average steady state Nusselt number on the disk for Dirichlet condition (–)
R	radius of the cavity (m)
Ra_T	Rayleigh number for Dirichlet condition (–)
Ra_φ	Rayleigh number for Neumann condition (–)
S_h	total exchange area of the disk (m^2)
S_i	area of the ith element of the hot wall (m^2)
t	time (s)
T	temperature (K)
T_c	dome (cold wall) and ambient environment temperature (K)
T_h	disk temperature (hot wall) (K)
T^*	dimensionless temperature (–)
$U, U_{\max}$	velocity and its maximum value ($m\ s^{-1}$)
U^*	dimensionless velocity (–)
u_r, u_ϕ, u_θ	velocity components in r,ϕ,θ directions respectively ($m\ s^{-1}$)
x,y,z	Cartesian coordinates (reference frame tied to the inclined hemisphere)
x',y',z'	Cartesian coordinates (fixed reference frame)
z^*	dimensionless coordinate normal to the disk plane; $z^* = z/R$ (–)

Greek symbols

α	inclination angle of the disk (°)
β	volumetric expansion coefficient (K^{-1})
$\delta_{T-\varphi}(\alpha)$	deviation between $C_\varphi(\alpha)$ and $C_T(\alpha)$ (%)
φ	heat flux imposed to the disk for Neumann condition (Wm^{-2})
λ	thermal conductivity ($Wm^{-1}\ K^{-1}$)
μ	dynamic viscosity (Pa s)
ρ	density ($kg\ m^{-3}$)
ϕ,θ	angular and spherical coordinates (–)

The applications of transient natural convection in hemispherical cavities are mostly related to surveillance and security systems. This field widely uses hemispherical sensors whose active parts are electronic circuits contained in disks that are inclined with respect to horizontal plane at an angle depending on the area to be controlled. These sensors often operate by being alternately powered on and off. To ensure their reliability, their thermal state must be controlled throughout their operation in transient regime and in steady state. Transient convective heat transfer is quantified by means of correlations of Nusselt–Rayleigh–Fourier type corresponding to the disk in horizontal position (with an upwards-oriented dome), vertical position or inclined with respect to the horizontal plane. The enclosures inclined at angles $0° \le \alpha \le 90°$ represented in Fig. 1 are treated for the condition of heat flux imposed on the disk in [18], and for imposed temperature on the disk in [19]. The Rayleigh number ranges corresponding to these studies are $10^4 \le Ra_\varphi \le 3.2 \times 10^{11}$ and $10^4 \le Ra_T \le 2.55 \times 10^{12}$ respectively. Relationships of Nusselt–Rayleigh–Fourier type dealing with $90° \le \alpha \le 180°$ step 10° are considered in [20] for the Neumann condition, valid for $10^4 \le Ra_\varphi \le 5 \times 10^8$.

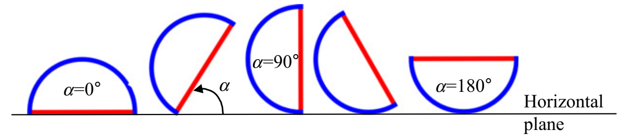

Fig. 1. Some representative inclinations of the hemispherical cavity.

The transient convective heat transfer related to the imposed temperature condition on the inclined disk has not been quantified in previous works. This motivates the present survey which completes the previous results on the transient convective heat transfer in the hemispherical cavities. The relationships proposed in this survey are valid for enclosures whose disk is inclined with respect to the horizontal plane at an angle varying between 90° (vertical disk) and 180° (horizontal disk with dome downwards) by steps of 10°. The wide considered Rayleigh range varying between 10^4 and 5×10^8 allows applying them in several engineering fields such as nuclear technology, solar energy, security and safety, electronics, building, domotics or aeronautics.

2. The treated cavity

The objective of this work is to provide correlations of Nusselt–Rayleigh–Fourier type that allow quantifying the transient convective heat transfer that occurs in the air-filled hemispherical cavities of radius R sketched in Fig. 2(a). The entire cavity including the fluid (air), the dome and the disk are initially at ambient temperature T_c. The disk is suddenly heated and maintained isothermal at temperature T_h on its internal face, while its rear face remains thermally insulated. The disk is either vertical ($\alpha = 90°$) or inclined with respect to the horizontal plane at an angle varying between 90° and 180° (horizontal disk with dome downwards) by steps of 10°, as represented in Fig. 2(c). The imposed temperature difference between the disk and the dome, associated with the radius of the cavity, involves Rayleigh number Ra_T varying between 10^4 and 5×10^8.

3. Governing equations. Results

In the (r,θ,ϕ) coordinates represented in Fig. 2(c), the velocity and gravity components are denoted by u_r, u_θ, u_φ and g_r, g_θ, g_ϕ respectively. Being μ, ρ, C_p and $a = \lambda/\rho C_p$ the dynamic viscosity, density, specific heat at constant pressure and thermal diffusivity of the air respectively, the governing equations in the spherical coordinates are

continuity equation

$$\frac{\partial \rho}{\partial t} + \frac{1}{r^2}\frac{\partial}{\partial r}\left(\rho r^2 u_r\right) + \frac{1}{r\sin\theta}\left[\frac{\partial\left(\rho u_\phi\right)}{\partial \phi} + \frac{\partial}{\partial \theta}\left(\rho u_\theta \sin\theta\right)\right] = 0 \qquad (1)$$