



## Two approaches to modelling the heating of evaporating droplets<sup>☆</sup>



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### ARTICLE INFO

Available online 13 August 2014

#### Keywords:

Droplet heating  
Droplet evaporation  
n-Dodecane  
Heat/mass transfer

### ABSTRACT

Two approaches to modelling the heating of evaporating droplets have been widely used in engineering applications. In the first approach the heat rate supplied to the droplets to raise their temperatures,  $\dot{q}_d$ , is derived from the requirement that droplet evaporation rates, inferred from steady-state equations for mass and heat balance, should be the same. The second approach is based on the direct calculation of the distribution of temperature inside droplets assuming that their thermal conductivity is not infinitely large. The implications of these two approaches are compared for the case of stationary droplets in conditions relevant to Diesel engines. It is pointed out that although the trends of time evolution of  $\dot{q}_d$  predicted by both approaches are similar, actual values of  $\dot{q}_d$  predicted by these approaches can be visibly different. This difference can lead to noticeable differences in predicted droplet surface temperatures, radii and evaporation times. Possible reasons for these differences are discussed.

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### 1. Introduction

Since the pioneering publications by Spalding (see [1]), the evaporation rate of stationary droplets  $\dot{m}_d$  has been estimated based on the following well known equation [2,3]:

$$\dot{m}_d = -4\pi R_d D_v \rho_{\text{total}} \ln(1 + B_M), \quad (1)$$

where

$$B_M = \frac{\rho_{vs} - \rho_{v\infty}}{\rho_{gs}} \quad (2)$$

is the Spalding mass transfer number,  $\rho_{vs}$  and  $\rho_{v\infty}$  are densities of vapour in the vicinity of droplet surfaces and at a large distance from them,  $\rho_g$  is the density of the ambient gas (air),  $R_d$  is the droplet radius,  $D_v$  is the diffusion coefficient of vapour in gas, and  $\rho_{\text{total}} = \rho_g + \rho_v$  is the total density of the mixture of vapour and gas. Note that  $\dot{m}_d \leq 0$ .

The derivation of Expression (1) was based on a number of assumptions, perhaps one of the most important of which is the assumption that  $\rho_{\text{total}} = \text{const}$  and does not depend on the distance from the droplet surface. This assumption can be justified when the temperature of the droplet is low and the difference between gas and droplet surface temperature is small (slow evaporation). In practical engineering applications, however, these restrictions of the range of applicability of Expression (1) are commonly ignored (e.g. [4]). Note that Expression (1)

cannot be used when the droplet surface temperature approaches boiling temperature when  $\rho_{gs} = 0$  and  $B_M$  becomes infinitely large (e.g. [5]).

A more rigorous approach to the analysis of droplet evaporation was presented by Tonini and Cossali [6,7]. In the model suggested and developed in these papers, the requirement that  $\rho_{\text{total}} = \text{const}$  was relaxed. The species, momentum and energy conservation equations were solved in a radial coordinate system, accounting explicitly for the gas density dependence on temperature and vapour concentration. However, as in the case of Expression (1), the problem was solved based on the quasi-steady state approximation (terms proportional to partial time derivatives were ignored in all equations) and the droplet surface temperature was assumed to be fixed during any time step. The effects of temperature gradient inside droplets were ignored (their thermal conductivity was assumed to be infinitely large).

An alternative expression for  $\dot{m}_d$  was obtained based on the analysis of the heat balance equation. Assuming that the evaporating droplet is stationary, as in the case of Expression (1), this equation for an arbitrary distance  $R > R_d$  from the centre of the droplet can be presented in the form [8]:

$$4\pi R^2 k_g \frac{dT}{dR} = -\dot{m}_d c_{pv}(T - T_s) - \dot{m}_d L(T_s) + \dot{q}_d, \quad (3)$$

where  $k_g$ ,  $c_{pv}$  and  $L(T_s)$  are gas thermal conductivity, vapour specific heat capacity at constant pressure and specific heat of evaporation at the droplet surface temperature  $T_s$ . As in Expression (1),  $\dot{m}_d \leq 0$ . The left hand side of this equation shows the heat supplied from the surrounding gas to the droplet. The first term on the right hand side shows the heat required to heat fuel vapour from  $T_s$  to  $T = T(R)$  (gas temperature

<sup>☆</sup> Communicated by W.J. Minkowycz.

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### Nomenclature

$B_M$	Spalding mass transfer number
$B_T$	Spalding heat transfer number
$c$	specific heat capacity
$D_v$	binary diffusion coefficient of vapour in air
$h$	convection heat transfer coefficient
$h_0$	parameter introduced in Expression (9)
$k$	thermal conductivity
$L$	latent heat of evaporation
$Le$	Lewis number
$\dot{m}_d$	evaporation rate
$\dot{q}_d$	heat rate
$q_n$	parameter introduced in Expression (9)
$R$	distance from the droplet centre
$R_d$	droplet radius
$t$	time
$T$	temperature

### Greek symbols

$\kappa_R$	parameter introduced in Expression (9)
$\lambda$	parameter defined by Eq. (10)
$\mu_0$	parameter introduced in Expression (9)
$\rho$	density
$\varphi$	parameter defined by Eq. (7)

### Subscripts

$d$	droplet
$e$	evaporation
$eff$	effective
$g$	gas
$p$	constant pressure
$s$	surface
$v$	vapour
$\infty$	at a large distance from the droplet

at the distance  $R$  from the centre of the droplet). The second and third terms on the right hand side show the heat spent on droplet evaporation and raising its temperature (when  $\dot{q}_d > 0$ ) respectively.

Rearranging this equation and its integration from  $T = T_s$  to  $T = T_g$  (ambient gas temperature) and from  $R = R_d$  to  $R = \infty$ , assuming that the temperature dependence of  $k_g$  and  $c_{pv}$  can be ignored, gives [8]:

$$\dot{m}_d = -\frac{4\pi k_g R_d}{c_{pv}} \ln(1 + B_T), \quad (4)$$

where

$$B_T = \frac{c_{pv}(T_g - T_s)}{L(T_s) - (\dot{q}_d / \dot{m}_d)} \quad (5)$$

is the Spalding heat transfer number. From Eqs. (1) and (4) follows the relation between  $B_T$  and  $B_M$  [8]:

$$B_T = (1 + B_M)^\varphi - 1, \quad (6)$$

where

$$\varphi = \left( \frac{c_{pv}}{c_{pg}} \right) \frac{1}{Le}, \quad (7)$$

$Le = k_g / (c_{pg} D_{vg} \rho_{total})$  is the Lewis number.

Eq. (5) can be rewritten as:

$$\dot{q}_d = -\dot{m}_d \left[ \frac{c_{pv}(T_g - T_s)}{B_T} - L(T_s) \right] = -\dot{m}_d \left[ \frac{c_{pv}(T_g - T_s)}{(1 + B_M)^\varphi - 1} - L(T_s) \right]. \quad (8)$$

Since the pioneering paper by Abramzon and Sirignano [8], Expression (8) has been widely used for modelling the heating of evaporating droplets. The assumptions on which the derivation of this expression was based (e.g. the validity of Expression (1)) have been almost universally ignored. An obvious limitation of Expression (8) is that the value of  $\dot{q}_d$  is not affected by the thermal conductivity of liquid, which contradicts the physical nature of  $\dot{q}_d$ , as discussed later in the paper.

An alternative approach to the calculation of  $\dot{q}_d$  could be based on the analysis of temperature distribution inside droplets, inferred from the direct analysis of convective heating of evaporating droplets (see [2] for the details). This approach is restricted to the case when liquid thermal conductivity is finite, which can be expected for any realistic liquid.

The focus of this paper is on the comparison of these two approaches to the calculation of  $\dot{q}_d$  and their implications for the modelling of droplet heating and evaporation in conditions typical for Diesel engines. The analysis is focused on stationary droplets, although it can be easily generalised to the case of moving droplets, using the effective thermal conductivity (ETC) model (see [2,3]).

## 2. Model

Assuming that the convection heat transfer coefficient  $h = \text{const}$ , the solution to the heat conduction equation inside droplets, assuming that all processes are spherically symmetric, can be presented as [2,3]:

$$T(R, t) = \frac{R_d}{R} \sum_{n=1}^{\infty} \left\{ q_n \exp[-\kappa_R \lambda_n^2 t] - \frac{\sin \lambda_n}{\|v_n\|^2 \lambda_n^2} \mu_0(0) \exp[-\kappa_R \lambda_n^2 t] - \frac{\sin \lambda_n}{\|v_n\|^2 \lambda_n^2} \int_0^t \frac{d\mu_0(\tau)}{d\tau} \exp[-\kappa_R \lambda_n^2 (t - \tau)] d\tau \right\} \sin \left[ \lambda_n \left( \frac{R}{R_d} \right) \right] + T_{\text{eff}}(t), \quad (9)$$

where  $\lambda_n$  values are solutions to the equation:

$$\lambda \cos \lambda + h_0 \sin \lambda = 0, \quad (10)$$

$$\|v_n\|^2 = \frac{1}{2} \left( 1 - \frac{\sin 2\lambda_n}{2\lambda_n} \right) = \frac{1}{2} \left( 1 + \frac{h_0}{h_0^2 + \lambda_n^2} \right),$$

$$q_n = \frac{1}{R_d \|v_n\|^2} \int_0^{R_d} \tilde{T}_0(R) \sin \left[ \lambda_n \left( \frac{R}{R_d} \right) \right] dR, \quad \kappa_R = \frac{k_l}{c_l \rho_l R_d^2}, \quad \mu_0(t) = \frac{h T_g(t) R_d}{k_l},$$

$h_0 = (h R_d / k_l) - 1$  and  $\tilde{T}_0(R) = R T_{d0}(R) / R_d$ . The solution to Eq. (10) gives a set of positive eigenvalues  $\lambda_n$  numbered in ascending order ( $n = 1, 2, \dots$ ),

$$T_{\text{eff}} = T_g + \frac{\rho_l L \dot{R}_{de}}{h}, \quad \dot{R}_{de} = \frac{\dot{m}_d}{4\pi R_d^2 \rho_l},$$

where  $\rho_l$  is the liquid density, and  $h$  for stationary evaporating droplets can be estimated as [2]:

$$h = \frac{2k_g}{R_d} \frac{\ln(1 + B_T)}{B_T}, \quad (11)$$

where  $B_T$  is defined by Eq. (5).

The rate of droplet heating, leading to the rise of their temperatures, can be estimated as

$$\dot{q}_d = 4\pi R_d^2 k_l \left. \frac{\partial T}{\partial R} \right|_{R=R_d-0} \quad (12)$$

where  $\dot{q}_d > 0$  when the droplet is heated.

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