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Nonlinear Rayleigh–Taylor instability of cylindrical flow with mass transfer through porous media ☆

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ABSTRACT

The work discusses nonlinear Rayleigh–Taylor instability of the interface between two viscous, incompressible and thermally conducting fluids in a fully saturated porous medium, when the phases are enclosed between two horizontal cylindrical surfaces coaxial with the interface, and when there is mass and heat transfer across the interface. We use viscous potential flow theory in which the flow is assumed to be irrotational and viscosity enters through normal viscous stresses at the interface. The perturbation analysis, in the light of the multiple expansions in both space and time, leads to imposing the well-known Ginzburg–Landau equation. The various stability conditions are discussed both analytically and numerically. The results are displayed in many plots showing the stability criteria in various parameter planes. It is observed that heat and mass transfer has stabilizing effect on the stability of the considered system while medium porosity destabilizes the interface. The flow through porous media is more stable than the pure flow.

1. Introduction

In recent years, the researchers show a great interest on the fluid flow through porous medium due to its importance in various fields such as in the fields of agriculture engineering to study the underground water resources, seepage of water in river beds and in petroleum technology. The Darcy's law relates the movement of fluid to the pressure gradients acting on a parcel of fluid. Sunil and Sharma [1] studied the Rayleigh–Taylor instability of partially ionized plasma in a porous medium in the presence of a variable horizontal magnetic field and uniform vertical rotation. They found that the medium permeability and rotation do not have any qualitative effect on the nature of the stability. Allah [2] considered the Rayleigh–Taylor instability of two fluids with exponential densities through porous media. El-Dib [3] investigated the nonlinear hydro-magnetic Rayleigh–Taylor stability of two viscous fluids in a porous medium and observed that the medium permeability plays a dual role in the stability analysis.

The instability of fluid flows in the presence of heat and mass transfer has been considered by many investigators because heat and mass transfer phenomena are encountered in a wide variety of engineering applications such as boiling heat transfer and geophysical problems. The effect of heat and mass transfer on the stability of the interface between two inviscid and incompressible fluids has been extensively studied in the past, and among the published studies are Hsieh [4,5] and Ho [6]. Khodaparast et al. [7] studied the linear stability analysis of a liquid–vapor interface but they considered liquid as viscous and motionless while vapor was inviscid moving with a horizontal velocity.

Awasthi and Agrawal [8] investigated the heat and mass transfer effects on the Rayleigh–Taylor instability of two viscous fluids and found that mass transfer effect stabilizes the interface. The heat transfer effect on the Kelvin–Helmholtz instability of miscible fluids using viscous potential flow theory was made by Asthana and Agrawal [9]. They observed that the heat and mass transfer has a strong stabilizing effect when the lower fluid is highly viscous and a weak destabilizing effect when the fluid's viscosity is low. Kim et al. [10] studied the capillary instability including the effect of interfacial heat and mass transfer and noted that the interfacial heat and mass transfer phenomenon resists the growth of disturbance waves.

The linear stability of fluid flows through porous media in the presence of heat and mass transfer using viscous potential flow theory has also been investigated by some authors in the recent years. Allah [11] considered the effect of porous medium on the interfacial instability with heat and mass transfer. The effect of porous medium on the capillary instability in the presence of heat and mass transfer was investigated by Awasthi and Asthana [12]. They found that unlike Kelvin–Helmholtz instability, porous medium plays a stabilizing role in the stability criterion.

The uniform model based on the linear theory is inadequate to explain the mechanism involved in the stability analysis because in this theory, second and higher order terms of perturbed quantities are

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neglected and therefore, the nonlinear theory is needed to study the mechanism behind the instability. Hsieh [13] considered the nonlinear Rayleigh–Taylor instability at the plane interface in the presence of heat and mass transfer and found that when there is strong heat and mass transfer across the interface, nonlinearity increases the stability range for Rayleigh–Taylor instability. Lee [14] studied the Rayleigh–Taylor instability at the cylindrical interface with heat and mass transfer using inviscid potential flow analysis. He concluded that heat transfer has no effect in the linear analysis while it stabilizes the interface in the nonlinear theory.

If the flow is irrotational, the viscous term in the Navier–Stokes equation is zero but the viscous stresses are not zero. In the viscous potential flow theory, viscosity enters through normal stress balance, and therefore, the nonlinear theory based on viscous potential flow analysis gives more efficient results as compared to the inviscid theory. Awasthi et al. [15] used viscous potential flow theory to study the nonlinear Rayleigh–Taylor instability at the plane interface of two viscous fluids when there is heat and mass transfer across the interface. The nonlinear capillary instability in the presence of heat and mass transfer was studied by Awasthi and Agrawal [16]. Recently, Awasthi [17] studied the viscous potential flow analysis of nonlinear Rayleigh–Taylor instability of cylindrical flow in a viscous medium in the presence of heat and mass transfer and found that the nonlinear theory reduces the stability of the system.

Although there is a large literature on the Rayleigh–Taylor instability with heat and mass transfer but most of the published work has considered either linear theory in viscous fluids or nonlinear analysis in inviscid fluids. However, the coupling of nonlinear effects with heat transfer has a direct effect in various applications such as boilers, condensers, reactors, and other industrial processes, and a nonlinear theory is essential to reveal the effect of heat and mass transfer on the stability of the system. Therefore, in the present article, the nonlinear Rayleigh–Taylor instability of the interface between two viscous, incompressible and miscible fluids in a fully saturated porous medium has been considered, when the phases are enclosed between two horizontal cylindrical surfaces coaxial with the interface, and when there is mass and heat transfer across the interface. The viscous potential flow theory has been employed and a third order nonlinear theory for the propagation of waves on the cylindrical interface has been developed. We have used the method of multiple scales for the investigation and the well-known Ginzburg–Landau equation describing the nonlinear waves has been obtained. In addition, a comparative analysis has been made between the results obtained in the viscous medium (Awasthi [17]) and the present analysis.

2. Problem formulation

We consider a system consisting of two incompressible, thermally conducting and viscous fluids, separated by a cylindrical interface $r = R$, in an annular porous medium with constant porosity ε and constant permeability k_1 as shown in Fig. 1. We consider a cylindrical system of

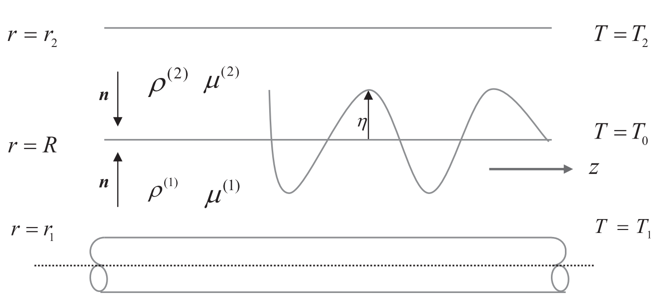

Fig. 1. Equilibrium configuration of the system.

coordinates (r, θ, z), so that in the equilibrium state z-axis is the axis of symmetry of the system. The inside fluid (1) occupies the inner region $r_1 < r < R$, having thickness h_1, density $\rho^{(1)}$ and viscosity $\mu^{(1)}$ and is bounded by the rigid cylindrical surface $r = r_1$ while the outside fluid (2) occupies the outer region $R < r < r_2$, having thickness h_2, density $\rho^{(2)}$ and viscosity $\mu^{(2)}$ and is bounded by the rigid cylindrical surface $r = r_2$, where $h_1 = R - r_1$ and $h_2 = r_2 - R$. The temperatures at $r = r_1$, $r = R$ and $r = r_2$ are T_1, T_0 and T_2, respectively and surface tension at the interface is taken as σ. We have assumed that both fluids are incompressible and irrotational. In the basic state, thermodynamics equilibrium is assumed and the interface temperature T_0 is set equal to the saturation temperature.

On applying the small disturbances to the equilibrium state, the interface can be expressed as

$$F(r, z, t) = r - R - \eta(z, t) = 0 \tag{2.1}$$

where η is the varicose interface displacement, and for which the outward unit normal vector is given by

$$\hat{\boldsymbol{n}} = \frac{grad\ F}{|grad\ F|} = \left\{1 + \left(\frac{\partial \eta}{\partial z}\right)^2\right\}^{-1/2}\left(\boldsymbol{e_r} - \frac{\partial \eta}{\partial z}\boldsymbol{e_z}\right) \tag{2.2}$$

where $\boldsymbol{e}_r$ and $\boldsymbol{e}_z$ are unit vectors along the r and z directions, respectively.

In this analysis, the Darcy's model has been used to include the effect of porous medium, therefore the equation governing the motion of viscous, incompressible fluids through porous medium can be written as

$$\frac{\rho}{\varepsilon}\left[\frac{\partial u}{\partial t} + \frac{1}{\varepsilon}(\boldsymbol{u} \cdot \nabla)\boldsymbol{u}\right] = -\nabla p - \frac{\mu}{k_1}\boldsymbol{u} \tag{2.3}$$

and equation of continuity

$$\nabla \cdot \boldsymbol{u} = 0. \tag{2.4}$$

Here p represents the pressure, μ denotes the fluid viscosity, k_1 is the medium permeability and ε represents the porosity of the medium which is defined as the fraction of the total volume of the medium that is occupied by void space.

We have considered that the motion is irrotational, so that the velocity can be expressed as the gradient of a potential function i.e.

$$\boldsymbol{u_j} = \nabla \phi^{(j)} \quad (j = 1, 2)\cdot \tag{2.5}$$

The potential functions satisfy the Laplace equation as a consequence of the incompressibility constraint. That is,

$$\nabla^2 \phi^{(j)} = 0, \quad (j = 1, 2)\cdot \tag{2.6}$$

3. Boundary conditions

The solutions for the potential functions $\phi^{(j)}, j = 1, 2$ should satisfy the following boundary conditions:

(1) The normal velocity vanishes at the rigid cylindrical surfaces $r = r_1$ and $r = r_2$, so we have

$$\frac{\partial \phi^{(j)}}{\partial r} = 0 \quad \text{at} \quad r = r_j, \tag{3.1}$$

(2) At the free interface $r = R + \eta(z,t)$, we have

(i) The interfacial condition, which expresses the conservation of mass across the interface, is given by the equation

$$\left[\rho\left(\frac{\partial F}{\partial t} + \nabla\phi \cdot \nabla F\right)\right] = 0 \tag{3.2}$$