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Three-dimensional viscous flow and heat transfer over a permeable shrinking sheet☆

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ABSTRACT

This paper presents a numerical analysis of a steady three-dimensional fluid flow and heat transfer towards a permeable shrinking sheet. The governing nonlinear partial differential equations are transformed into a system of ordinary differential equations by a similarity transformation, which are then solved numerically by a shooting method. The effects of the governing parameters on the skin friction and heat transfer from the surface of the shrinking surface are illustrated graphically. It is found that dual solutions exist for the shrinking case. A comparison with known results from the open literature has been done and it is shown to be in excellent agreement.

1. Introduction

Recently, the flow due to a stretching sheet has been studied rigorously because of its important applications in industries such as manufacturing of polymer sheets, filaments and wires. During the manufacturing process, the moving sheet is assumed to stretch on its own plane and the stretched surface interacts with the ambient fluid both mechanically and thermally (Crane [1], Gupta and Gupta [2], Carragher and Crane [3], etc.). More recently, the boundary layer flow due to a shrinking sheet has gained considerable interest. In contrast to stretching sheet, for the shrinking case, the sheet is shrunk towards a fixed point which would cause a velocity away from the sheet. This phenomenon can be found, for example, on a rising and shrinking balloon, or a moving and shrinking polymer film. Wang [4] was the first to study the unsteady two-dimensional viscous flow by shrinking film. The closed form exact solution of the viscous flow with suction has been obtained by Miklavčič and Wang [5] who found that the solutions may not be unique for certain suction rates. There are two conditions for shrinking flow to exist physically, i.e. either imposed adequate suction on the boundary (Fang et al. [6]) or added stagnation flow which contains the vorticity (Wang [7]). Some papers have also studied the case of three-dimensional stretching sheet (Wang [8], and Surma Devi et al. [9]).

The aim of this study is to investigate the steady three-dimensional flow and heat transfer past a permeable shrinking sheet. As far as we are concerned, the problem of the three-dimensional shrinking sheet has yet to be solved. After applying the boundary layer approximations and similarity transformation, the resulting nonlinear governing equations are solved numerically for some values of the governing parameters. Representative results for the skin friction coefficients and heat flux from the surface of the sheet are presented. To the best of our knowledge the results of this paper are original, new, very interesting and they have not been published before.

2. Basic equations

We consider the steady three-dimensional boundary layer flow of a viscous fluid past a permeable shrinking flat surface in an otherwise quiescent fluid. A locally orthogonal set of coordinates (x, y, z) is chosen with the origin O in the plane of the shrinking sheet. The $x-$ and $y-$ coordinates are in the plane of the shrinking sheet, while the coordinate z is measured in the perpendicular direction to the surface as shown in Fig. 1. It is assumed that the flat surface is shrunk continuously in both x- and $y-$ directions with the velocities $u(x) = u_w(x)$ and $v(y) = v_w(y)$, respectively. It is also assumed that the mass flux velocity is $w = w_0$, where $w_0 < 0$ is for suction and $w_0 > 0$ is for injection or withdrawal of the fluid. Under these conditions, the boundary layer equations are (Surma Devi et al. [9])

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}=\nu\frac{\partial^2 u}{\partial z^2} \tag{2}$$

$$u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z}=\nu\frac{\partial^2 v}{\partial z^2} \tag{3}$$

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Nomenclature

a	positive constant
C_{fx}, C_{fy}	local skin friction coefficients
$f(\eta), g(\eta)$	dimensionless functions
Nu_x	local Nusselt number
Pr	Prandtl number
Re_x, Re_y	local Reynolds numbers
s	mass transfer parameter
T	fluid temperature
u, v, w	velocity components along the x, y, z directions, respectively
$u_w(x), v_w(y)$	shrinking velocities in the x- and y-directions, respectively
$U_w(x), V_w(y)$	constant shrinking velocities in the x- and y-directions, respectively
w_0	constant mass flux velocity
x, y, z	Cartesian coordinates

Greek symbols

α	thermal diffusivity
η	similarity variable
λ, λ_1	shrinking parameters
ν	kinematic viscosity
τ_{wx}, τ_{wy}	skin frictions or shear stresses in the x- and y-directions, respectively

$$u\frac{\partial w}{\partial x}+v\frac{\partial w}{\partial y}+w\frac{\partial w}{\partial z}=\nu\frac{\partial^2 w}{\partial z^2} \tag{4}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}+w\frac{\partial T}{\partial z}=\alpha\frac{\partial^2 T}{\partial z^2} \tag{5}$$

along with the boundary conditions

$$\begin{aligned}&u=u_w(x)=\lambda U_w(x), \quad v=v_w(y)=\lambda_1 V_w(y), \quad w=w_w=w_0,\\ &T=T_w \quad \text{at} \quad z=0, u(x,y,z)\to 0, \quad v(x,y,z)\to 0, \quad w(x,y,z)\to 0,\\ &T\to T_\infty \quad \text{as} \quad z\to\infty\end{aligned} \tag{6}$$

Here u, v and w are the velocity components along the $x-$, $y-$ and $z-$ axes, respectively, T is the temperature of the fluid, T_w and T_∞ are the constant surface temperature and the constant ambient temperature, respectively, α is the thermal diffusivity of the fluid, ν is the kinematic viscosity of the fluid, and λ and λ_1 are the stretching ($\lambda, \lambda_1 > 0$) or shrinking ($\lambda, \lambda_1 < 0$) parameters. Here, we assume that $U_w(x) = ax$ and $V_w(y) = ay$, where a is a positive constant.

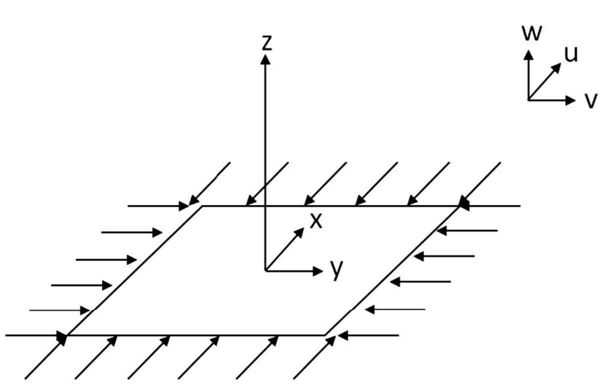

Fig. 1. Physical model and coordinate system.

Table 1
Comparison values for $-f''(0)$, $-g''(0)$ and $-\theta'(0)$ with those of Surma Devi et al. [9] by considering the steady state case, using the following parameters: Pr = 0.7, $s = 0$, $\lambda = 1$, $\lambda_1 = 0.5$, (stretching case) in boundary conditions (11).

Surma Devi et al. [9]			Present results		
$-f''(0)$	$-g''(0)$	$-\theta'(0)$	$-f''(0)$	$-g''(0)$	$-\theta'(0)$
1.0931	0.4652	0.5758	1.0931	0.4652	0.5757

3. Solutions

We introduce now the following similarity variables:

$$\begin{aligned}&u=axf'(\eta), \quad v=ayg'(\eta), \quad w=-(a\nu)^{1/2}[f(\eta)+g(\eta)]\\ &\theta(\eta)=(T-T_\infty)/(T_w-T_\infty), \quad \eta=(a/\nu)^{1/2}z\end{aligned} \tag{7}$$

where primes denote differentiation with respect to η. Substituting the variables (7) into Eqs. (1) to (5), it is found that the continuity Eq. (1) is automatically satisfied, and Eqs. (2)–(5) are reduced to the following ordinary (similarity) differential equations:

$$f'''+(f+g)f''-f'^2=0 \tag{8}$$

$$g'''+(f+g)g''-g'^2=0 \tag{9}$$

$$\theta''+\Pr(f+g)\theta'=0 \tag{10}$$

subject to the boundary conditions

$$\begin{aligned}&f(0)=s, \quad g(0)=0, \quad f'(0)=\lambda, \quad g'(0)=\lambda_1, \quad \theta(0)=1\\ &f'(\eta)\to 0, \quad g'(\eta)\to 0, \quad \theta(\eta)\to 0 \quad \text{as} \quad \eta\to\infty\end{aligned} \tag{11}$$

Here $s = -w_0/(a\nu)^{1/2}$ is the surface mass transfer parameter with $s > 0$ for suction and $s < 0$ for injection, respectively, and $\Pr = \nu/\alpha$ is the

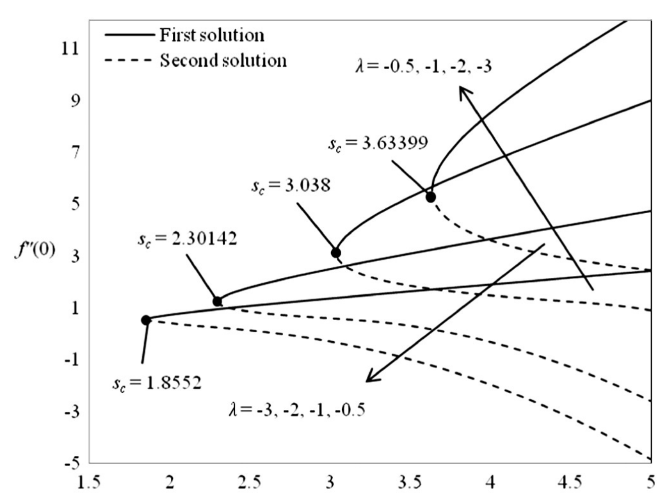

Fig. 2. Variation of $f''(0)$ with s for several values of $\lambda(<0)$ when $\lambda_1 = -0.5$.