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Interval inverse analysis of hyperbolic heat conduction problem ☆

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ABSTRACT

A general numerical model was presented to analyze the interval inverse hyperbolic heat conduction problem, with Bregman distances and weighted Bregman distances as regularization terms. By using the interval finite element method and interval extension theory, the direct and inverse models were established for uncertainties. The eight-point isoparametric elements were applied for the discretization in the space domain, and the Precise algorithm in time domain was empolyed. The inverse problems were implicitly formulated as optimization problems, using squared residues between the calculated and measured quantities as the objective function of the inverse identification. Results show that the proposed numerical models can identify single and combined interval thermal parameters and boundary conditions for hyperbolic heat transfer problems accurately and efficiently.

1. Introduction

Inverse heat conduction problems (IHCPs) arise in many engineering fields. Over the last decades, various analytical and numerical methods were developed to identify the single and combined variables for the elliptical and parabolic IHCP [1–4]. However, the reports on hyperbolic IHCP are few [5–7], and most of them are concentrated on single variable. The repeated forward analyses are required for the inverse calculation, so the accuracy of forward calculation directly determines the inverse results. Compared with the parabolic heat conduction problems, the hyperbolic direct and inverse problems are more complicated due to the existence of second order derivative of temperature with respect to time.

In previous studies, the parameters of IHCP were considered as deterministic variables and the parameter identification depended on the certain measurement information. They belong to deterministic inverse identification, such as the identification of heat transfer coefficient, boundary condition, and the source term. It is well known that there exist a lot of uncertainties in structural parameters caused by various sources, e.g. variability in material property, initial manufacturing errors, and aging deterioration of performance. These uncertainties may lead to various unexpected situations where the structural responses such as deformation and stress may exceed the performance limit. Hence, it is necessary to evaluate the uncertainty of the structural response more accurately and efficiently [8].

There are mainly three approaches to describe the uncertainty, including probabilistic method, fuzzy theory, and interval analysis. The input data of probabilistic description of uncertainties are described as random variables, or stochastic processes. In order to apply probabilistic method, one needs to use a large number of inputs to describe probability density functions of uncertain variables, functions, or fields. When such information is available, probabilistic method is a viable procedure to predict structural reliability, or its complement, probability of failure. The presence of only small deviations from the real probabilistic distributions may lead to significant errors in the final results [9–11]. Fuzzy theory has been successfully used in domains in which information is incomplete or imprecise, such as linguistic expressions, but the speed of information processing is low [12]. Interval analysis is based on interval operations including interval arithmetic. The technique calculates the interval between the upper and lower bounds regarding variables under uncertainty [13,14]. It is particularly useful when statistical information is not sufficient to describe the probability distribution of the uncertain parameters, or only the range of the uncertain parameters is known, and the response of the interval range is desirous. In the past few years, some reports have been developed for the interval conduction problems [15,16]. The reports on interval IHCP are few, especially for uncertain interval inverse problems of combined variables. The report of interval inversion for hyperbolic heat conduction problem has not appeared.

Inverse problems are typically ill-posed problems, which can be solved as optimization problems. Tikhonov regularization, named for

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Andrey Tikhonov, is one of the most effective regularization methods to deal with these problems [17]. As indicated by SilvaNeto and Cella [18], the proper choice of the regularization parameter is of key importance for the implementation of regularization methods for the solution of inverse problems. In order to achieve the purpose of noise suppression simultaneously, the Bregman distances [19,20] as an option to Tikhonov have been applied to regularization algorithm in this paper. Cidade et al. [21] used Bregman distances as Tikhonov's regularization terms for the restoration of atomic force microscopy nanoscale images; Pinheiro et al. [22] dealt with an inverse problem of radiative property estimation based on this regularization technique; Wang and Wang [20] developed two stage Bregman regularization homotopy inversion algorithm for the parameter estimation for metabolic networks. Although Bregman distances have been used in many fields, it seems that there is no relevant work on the use of this technique in the uncertain inversion field.

In this paper an uncertain interval inverse model was established for hyperbolic heat conduction problem with interval parameters. The direct model was obtained, using the Precise algorithm in time domain [23] and interval extension theory [24]. Using Bregman distances and weighted Bregman distances for regularization function, the inverse model was explored for uncertainty problem, and the numerical tests were given. The results indicate that the proposed model can identify interval parameters for hyperbolic IHCP with high computational precision and efficiency.

2. Governing equation

The governing equation, relevant boundary and initial conditions for the hyperbolic heat transfer problems can be written in tensor forms [25]

$$\tau\left[\boldsymbol{c}\boldsymbol{T}_{,t}\right]_{,t} + \boldsymbol{c}\boldsymbol{T}_{,t} = \left[\boldsymbol{k}_{ij}\boldsymbol{T}_{,j}\right]_{,i} + \boldsymbol{Q} \qquad \boldsymbol{x}_i \in \boldsymbol{\Omega} \tag{1}$$

$$\boldsymbol{T}(t) = \overline{\boldsymbol{T}}(t) \qquad \boldsymbol{x}_i \in \boldsymbol{\Gamma}_1 \tag{2}$$

$$\begin{aligned}\boldsymbol{n}_i\left(\boldsymbol{k}_{ij}T_{,j}\right) &= \boldsymbol{q}(t) + \boldsymbol{h}(t)\cdot(\boldsymbol{T}(t) - \boldsymbol{T}_a(t)) \\ &= \boldsymbol{q}(t) + \boldsymbol{h}(t)\cdot\boldsymbol{T}(t) - \boldsymbol{h}_a(t) \qquad \boldsymbol{x}_i \in \boldsymbol{\Gamma}_2\end{aligned} \tag{3}$$

$$\boldsymbol{T} = \boldsymbol{T}_0, \quad \boldsymbol{T}_{,t} = \boldsymbol{D}\boldsymbol{T}_0 \qquad t = 0 \tag{4}$$

where $\boldsymbol{T}$ denotes the temperature, τ represents a relaxation time, $\boldsymbol{c}$ and $\boldsymbol{k}_{ij}$ are the thermal parameters, $\boldsymbol{Q}$ is the strength of heat source and $\boldsymbol{T}_{,t}$ is the first order derivative of temperature with respect to time. $\boldsymbol{\Gamma} = \boldsymbol{\Gamma}_1 + \boldsymbol{\Gamma}_2$ represents the boundary of the domain, $\boldsymbol{n}_i$ refers to the unit vector of outside normal. $\boldsymbol{x}_i$ is the vector of coordinates, and $\boldsymbol{\Omega}$ represents the domain of the problem. Summation convention is applied to indexes j and i, which will cover either from 1 to 2 for 2D problems, or from 1 to 3 for 3D problems.

In the governing equation, boundary and initial conditions, the parameters are interval parameters, such as $\boldsymbol{k}_{ij}$, $\boldsymbol{c}$, $\boldsymbol{q}$, $\boldsymbol{h}$, $\boldsymbol{T}_a$, $\boldsymbol{Q}$ and $\boldsymbol{T}_0$. So the above equations are changed to differential equations with interval parameters.

3. Direct models

Due to the presence of the second derivative of temperature with respect to time on control equation, the differential equations are very difficult. A precise discrete algorithm in time domain [23] is applied in this paper. Specific process is as follows.

Within a discrete time interval, $\boldsymbol{T}$ is expanded as

$$\boldsymbol{T} = \sum_{m=0} \boldsymbol{T}^m s^m \tag{5}$$

where s is defined by

$$s = \frac{t - t_0}{t_s} \tag{6}$$

where t_0 and t_s represent the beginning time and the size of the time interval, respectively, and $\boldsymbol{T}^m$ is the expanding coefficient of $\boldsymbol{T}$ with order m.

By utilizing $\frac{d}{dt} = \frac{d}{ds}\frac{ds}{dt} = \frac{1}{t_s}\frac{d}{ds}$, the first and second derivatives of $\boldsymbol{T}$ with respect to t can be transformed into

$$\boldsymbol{T}_{,t} = \sum_{m=0} \frac{(m+1)}{t_s}\boldsymbol{T}^{m+1}s^m, \quad \left[\boldsymbol{T}_{,t}\right]_{,t} = \sum_{m=0} \frac{(m+1)(m+2)}{t_s^{\,2}}\boldsymbol{T}^{m+2}s^m. \tag{7, 8}$$

The other parameters of control equation and boundary conditions are expanded in the same way

$$\boldsymbol{k}_{ij} = \sum_{m=0} \boldsymbol{k}_{ij}{}^m s^m \quad \boldsymbol{c} = \sum_{m=0} \boldsymbol{c}^m s^m \quad \overline{\boldsymbol{T}} = \sum_{m=0} \overline{\boldsymbol{T}}^m s^m \tag{9–11}$$

$$\boldsymbol{q} = \sum_{m=0} \boldsymbol{q}^m s^m \quad \boldsymbol{h} = \sum_{m=0} \boldsymbol{h}^m s^m \quad \boldsymbol{T}_a = \sum_{m=0} \boldsymbol{T}_a{}^m s^m. \tag{12–14}$$

Substituting Eqs. (7)–(14) into Eqs. (1)–(4) and comparing the coefficient term of s^m, we can obtain

$$\begin{aligned}&\tau\sum_{m=0}^{N}\frac{(m+1)(m+2)}{t_s^{\,2}}\boldsymbol{c}^{N-m}\boldsymbol{T}^{m+2} + \tau\sum_{m=0}^{N}\frac{(m+1)^2}{t_s^{\,2}}\boldsymbol{c}^{N-m+1}\boldsymbol{T}^{m+1} \\ &+ \sum_{m=0}^{N}\frac{(m+1)}{t_s}\boldsymbol{c}^{N-m}\boldsymbol{T}^{m+1} = \sum_{m=0}^{N}\left[\boldsymbol{k}_{ij}^{N-m}\boldsymbol{T}_{,j}{}^{m}\right]_{,i} + \boldsymbol{Q}^N\end{aligned} \tag{15}$$

with the boundary conditions

$$\boldsymbol{T}^N = \overline{\boldsymbol{T}}^N \tag{16}$$

$$\boldsymbol{n}_i\sum_{l=0}^{N}\boldsymbol{k}_{ij}^{N-l}\boldsymbol{T}_{,j}^{l} = \boldsymbol{f}^N. \tag{17}$$

The interval finite element recursive format in the time domain can be obtained, using the interval analysis method based on element and interval parameters.

$$\begin{aligned}&\tau\sum_{m=0}^{N}\frac{(m+1)(m+2)}{t_s^{\,2}}\boldsymbol{C}^{N-m}(\boldsymbol{\varphi})\{\boldsymbol{T}\}^{m+2} + \tau\sum_{m=0}^{N}\frac{(m+1)^2}{t_s^{\,2}}\boldsymbol{C}^{N-m+1}(\boldsymbol{\varphi})\{\boldsymbol{T}\}^{m+1} \\ &+ \sum_{m=0}^{N}\frac{(m+1)}{t_s}\boldsymbol{C}^{N-m}(\boldsymbol{\varphi})\{\boldsymbol{T}\}^{m+1} + \sum_{m=0}^{N}\boldsymbol{K}^{N-m}(\boldsymbol{\varphi})\{\boldsymbol{T}\}^m = \boldsymbol{F}^N(\boldsymbol{\varphi})\end{aligned} \tag{18}$$

where $\boldsymbol{\varphi}$ represents the interval vector, which can be decomposed and expressed as $\boldsymbol{\varphi} = \boldsymbol{\varphi}^c + [-\Delta\boldsymbol{\varphi},\Delta\boldsymbol{\varphi}] = \boldsymbol{\varphi}^c + \Delta\boldsymbol{\varphi}\boldsymbol{e}$. $\boldsymbol{C}$,$\boldsymbol{K}$ and $\boldsymbol{F}$ are the heat capacity matrix, the stiffness matrix and the equivalent load, respectively. They can be expressed as follows

$$\begin{aligned}\boldsymbol{C}^{N-m}(\boldsymbol{\varphi}) &= \boldsymbol{C}^{N-m}(\boldsymbol{\varphi}^c) + \Delta\boldsymbol{C}^{N-m}, \quad \boldsymbol{K}^{N-m}(\boldsymbol{\varphi}) \\ &= \boldsymbol{K}^{N-m}(\boldsymbol{\varphi}^c) + \Delta\boldsymbol{K}^{N-m}, \quad \boldsymbol{F}^N(\boldsymbol{\varphi}) = \boldsymbol{F}^N(\boldsymbol{\varphi}^c) + \Delta\boldsymbol{F}^N\end{aligned} \tag{19}$$